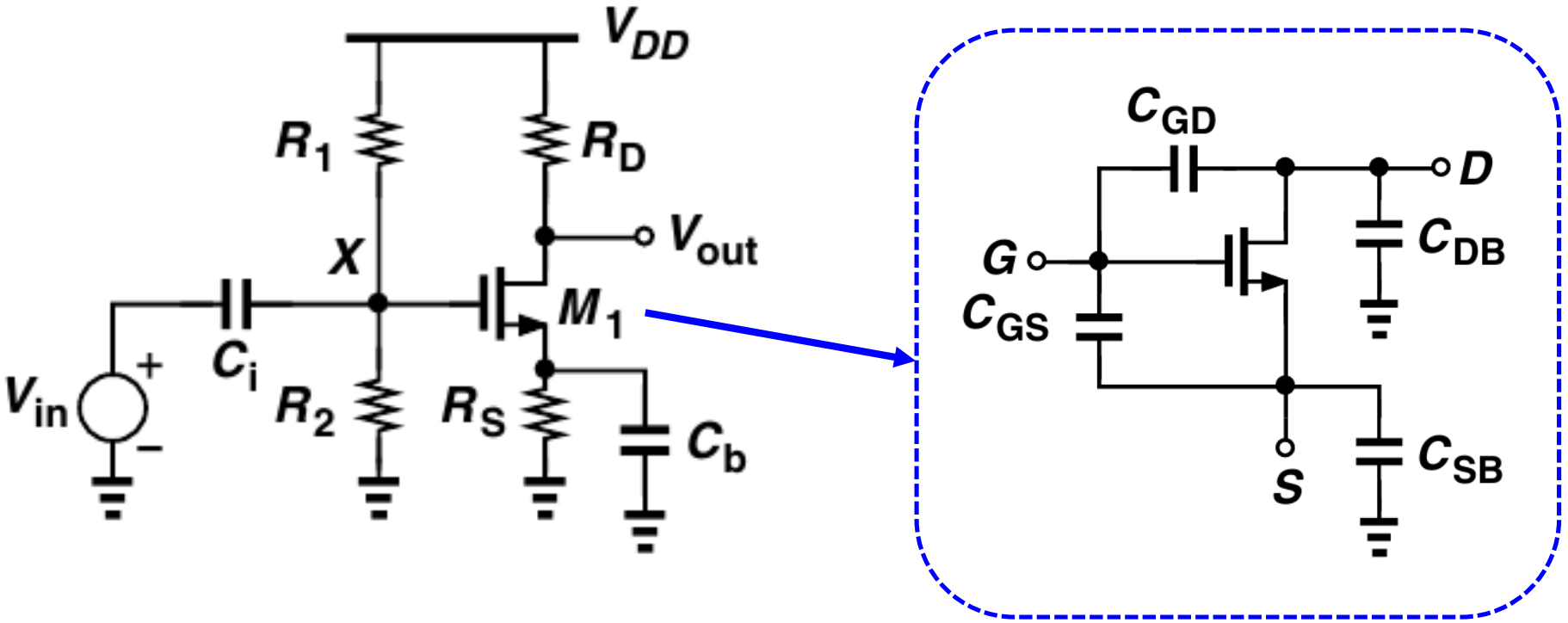
Frequency Response of CS Stage

Hanwool Jeong

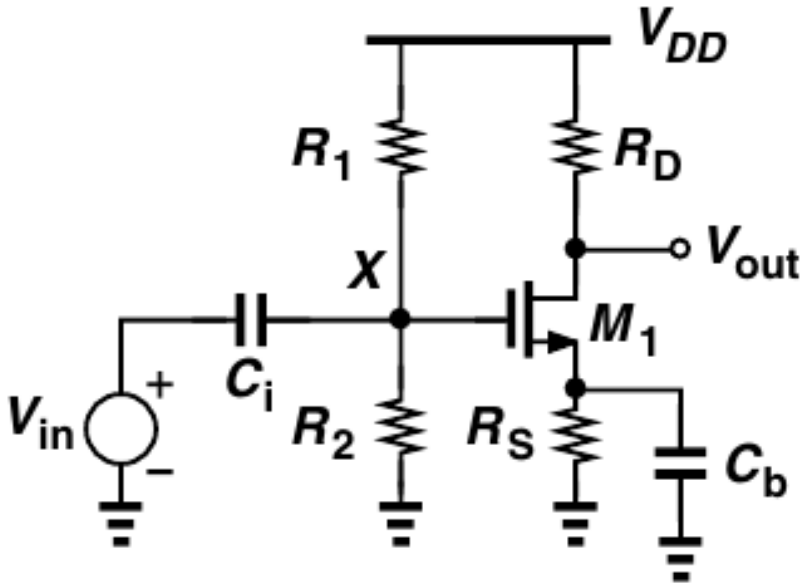
hwjeong@kw.ac.kr

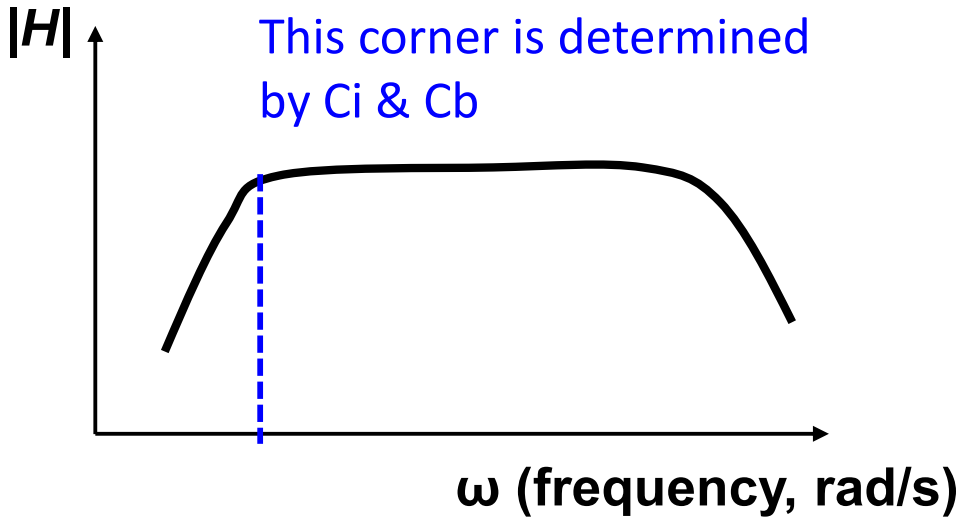
Capacitors in CS Stage



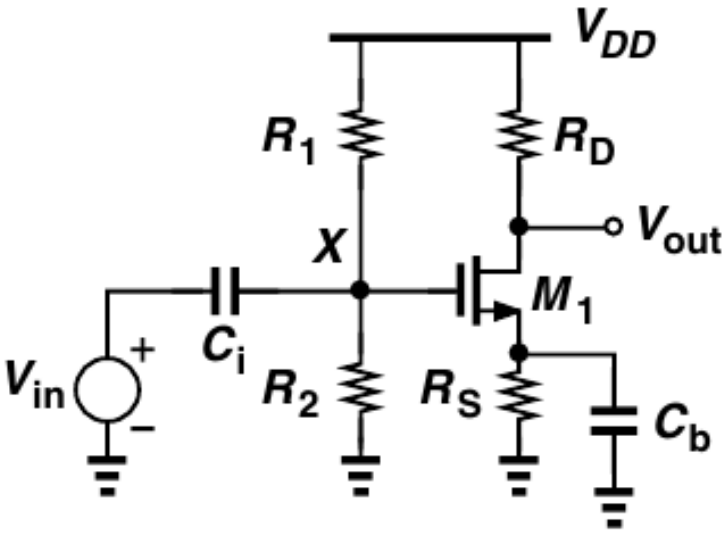
You Remember Role of C_i and C_b?

- Which is desirable? Large C_i vs. small C_i? How about C_b?
- Intuitively, we can say
  - Ci and Cb should be sufficiently large.
  - Ci and Cb determine the low frequency response.





Gain Dependence on C_i and C_b



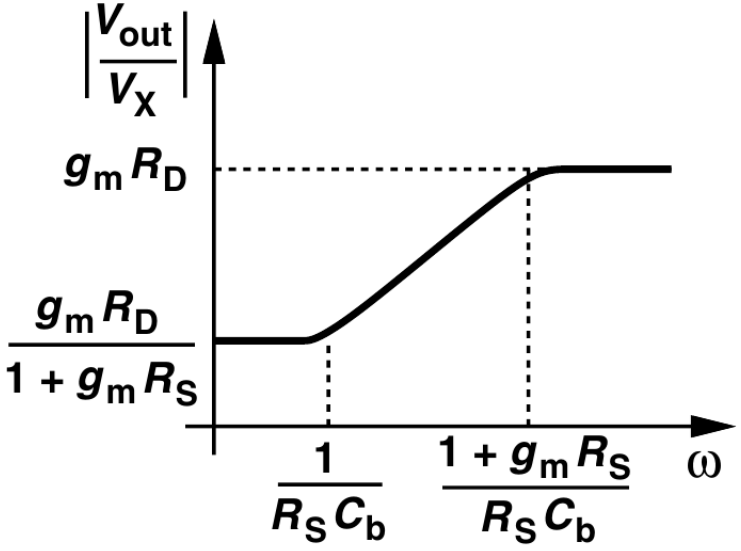
$$V_{out}/V_{in} = (V_{out}/V_X)(V_X/V_{in})$$

$$\frac{V_X}{V_{in}}(s) = \frac{R_1||R_2}{R_1||R_2 + \frac{1}{C_i s}} = \frac{(R_1||R_2)C_i s}{(R_1||R_2)C_i s + 1}$$

$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S||\frac{1}{C_b s} + \frac{1}{g_m}}$$

$$= \frac{-g_m R_D(R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

Cb Effect on Frequency Response of Through V_{out}/V_X

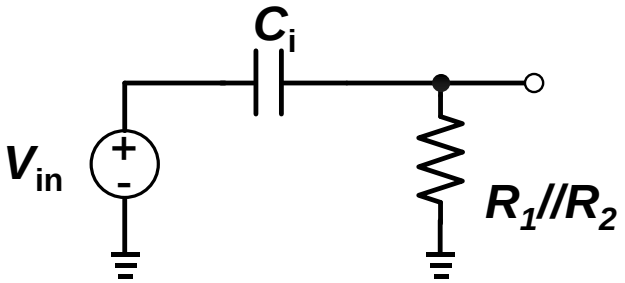


$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S || \frac{1}{C_b s} + \frac{1}{g_m}}$$

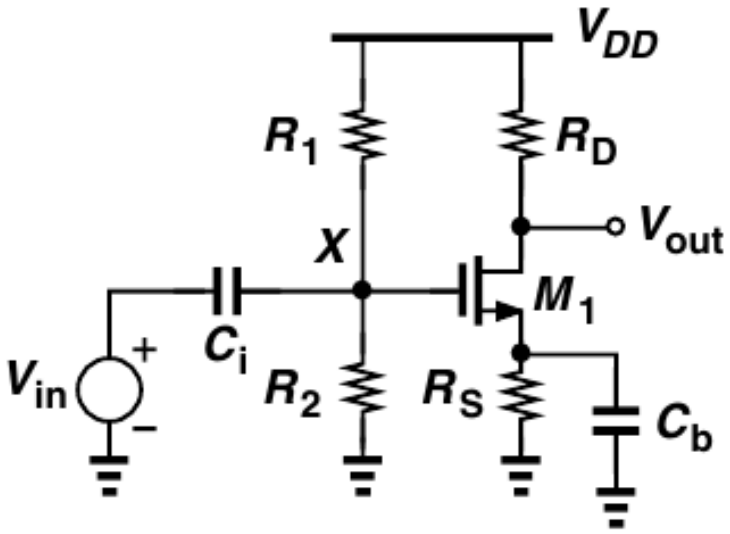
$$= \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

C_b and C_i Effects on Frequency Response

- Revisit Ci effect on V_X/V_{in}:



$$\frac{(R_1||R_2)C_i s}{(R_1||R_2)C_i s + 1}$$



- Combining Ci and Cb effects, we can form

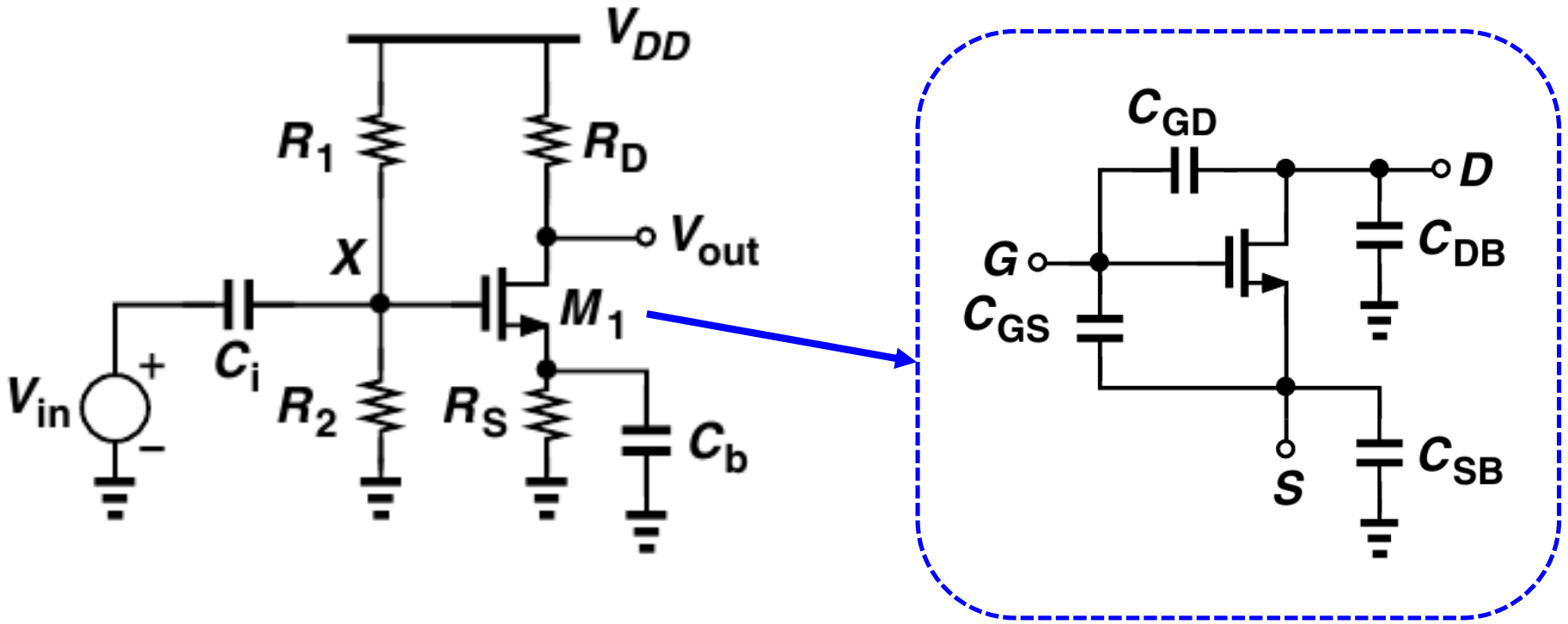
$$\frac{j\omega(R_1 \,||\, R_2)C_i}{1+ j\omega(R_1 \,||\, R_2)C_i} \times \frac{\dfrac{-g_m R_D}{1+g_m R_S}(1+ j\omega R_S C_b)}{1+ j\omega \dfrac{R_S C_b}{1+g_m R_S}}$$



Revisit Capacitors in CS Stage

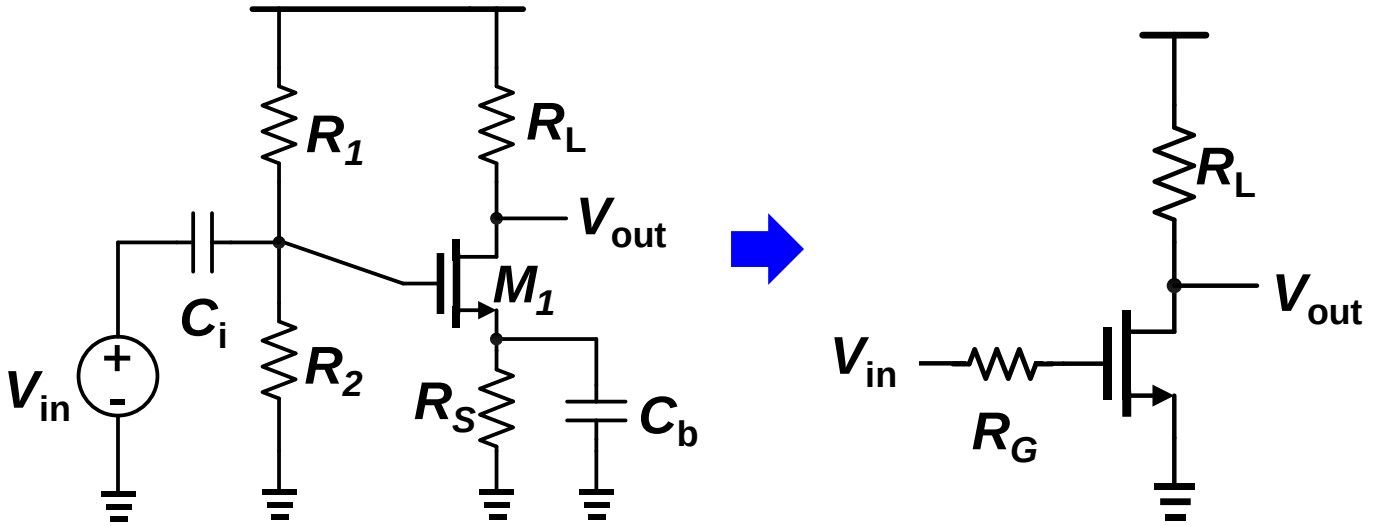
- Unavoidably exist in the transistors (relatively small)
- Load cap possibly exist due to the next stage.

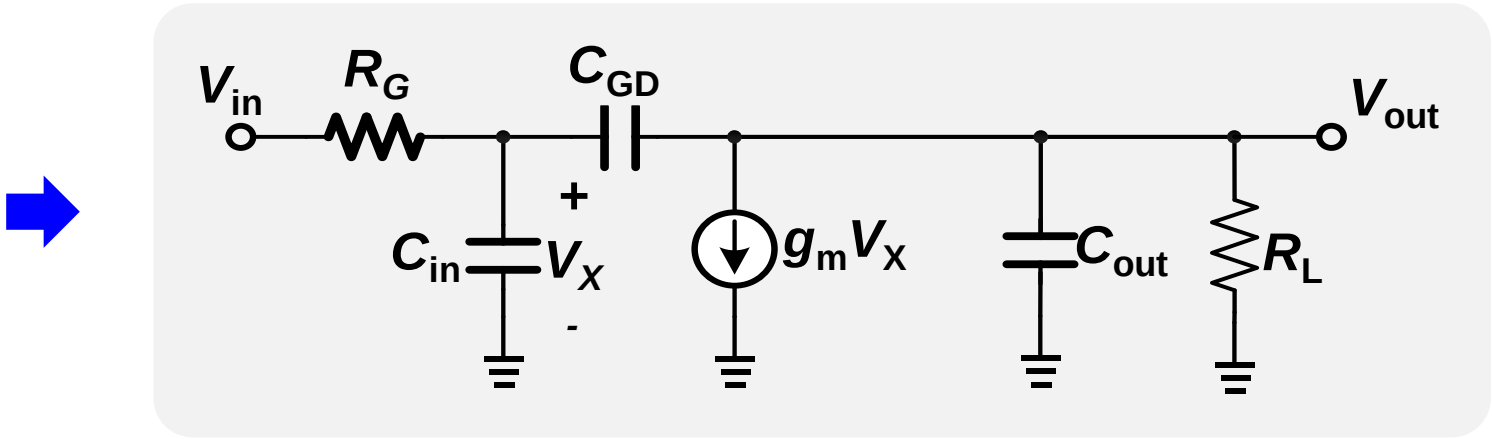
➔ Determines the high frequency response.



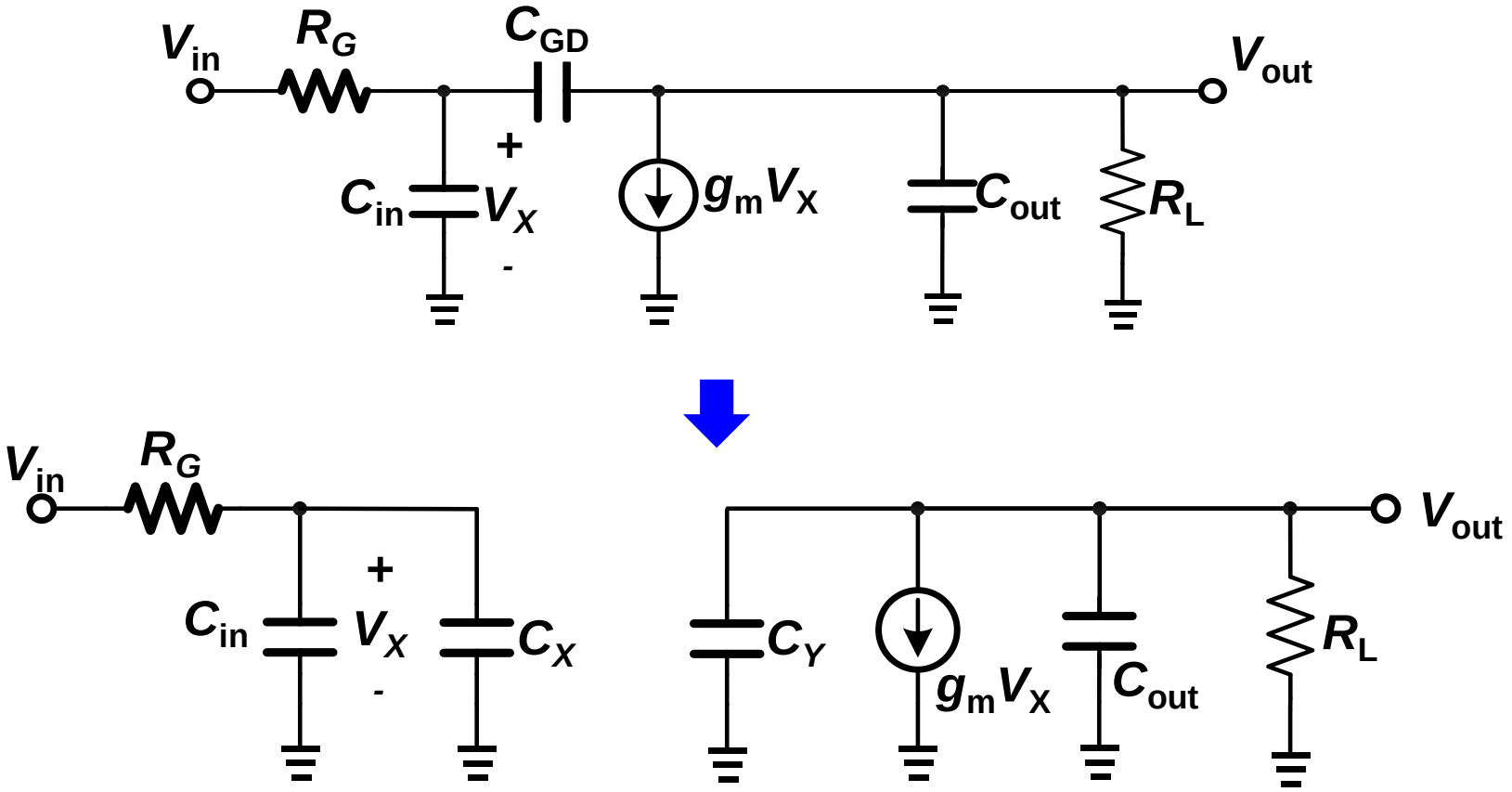
High Frequency Response of CS Stage

- As frequency is higher,





Use of Miller's Theorem

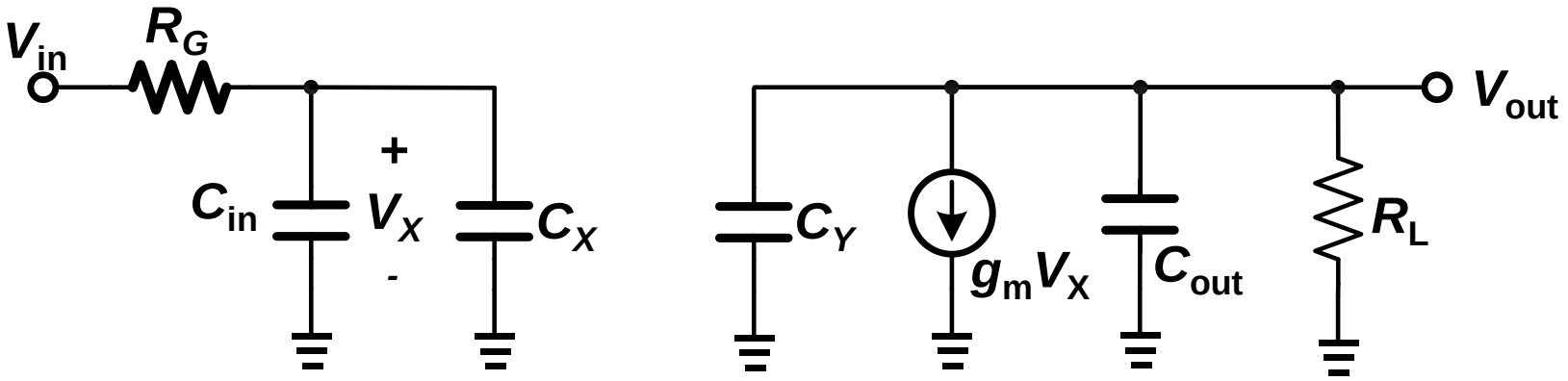


$$C_X = C_{GD}\,(1 + g_m R_L) \qquad C_Y = C_{GD}\,\left(1 + \frac{1}{g_m R_L}\right)$$

Use of Miller's Theorem
High Freq Poles in CS stage

$$\omega_{p,in} = \frac{1}{R_G[C_{in} + (1 + g_m R_L)C_{GD}]}$$

$$\omega_{p,out} = \frac{1}{R_L[C_{out} + (1 + \frac{1}{g_m R_L})C_{GD}]}$$

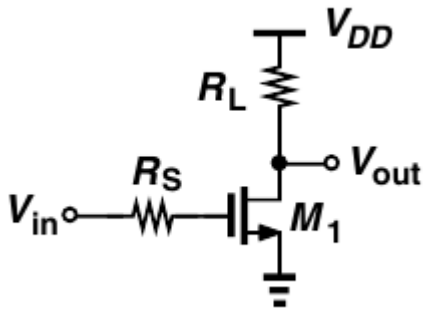


$$C_X = C_{GD}\,(1 + g_m R_L)$$

$$C_Y = C_{GD}\,(1 + \frac{1}{g_m R_L})$$

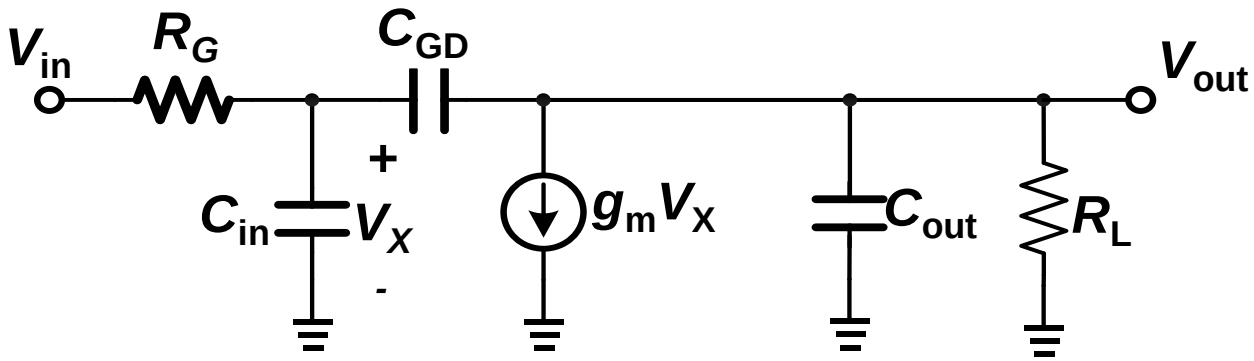
Example 11.16

An electrical engineering student designs the CS stage of Fig. 11.29(a) for a certain low-frequency gain and high-frequency response. Unfortunately, in the layout phase, the student uses a MOSFET half as wide as that in the original design. Assuming that the bias current is also halved, determine the gain and the poles of the circuit.



Direct Analysis

- High frequency ➔ Miller approximation is inaccurate.



- Using KCL,

$$(V_{out} - V_X)sC_{GD} = V_X sC_{in} + \frac{V_X - V_{in}}{R_G} \quad \text{(Eq. 1)}$$

$$(V_X - V_{out})sC_{GD} = g_m V_X + V_{out} sC_{out} + \frac{V_{out}}{R_L} \quad \text{(Eq. 2)}$$

$$(V_X - V_{out})sC_{GD} = g_m V_X + V_{out}sC_{out} + \frac{V_{out}}{R_L} \quad \text{(Eq. 2)}$$

$$\Rightarrow \quad (sC_{GD} - g_m)V_X = V_{out}s(C_{GD} + C_{out}) + \frac{V_{out}}{R_L}$$

$$\Rightarrow \quad V_X = V_{out}\frac{s(C_{GD} + C_{out}) + \frac{1}{R_L}}{sC_{GD} - g_m} \quad \text{(Eq. 3)}$$

$$(V_{out} - V_X)sC_{GD} = V_X sC_{in} + \frac{V_X - V_{in}}{R_G} \quad \text{(Eq. 1)}$$

$$\Rightarrow \quad sC_{GD}V_{out} - V_X\left(sC_{GD} + sC_{in} + \frac{1}{R_G}\right) = \frac{-V_{in}}{R_G} \quad \text{(Eq. 4)}$$

- Combining (Eq.3) & (Eq.4),

$$\left\{sC_{GD} - \left(sC_{in} + sC_{GD} + \frac{1}{R_G}\right)\frac{s(C_{GD} + C_{out}) + \frac{1}{R_L}}{sC_{GD} - g_m}\right\}V_{out} = \frac{-V_{in}}{R_G}$$

Gain Through Direct Analysis

- Thus,

$$\frac{V_{out}}{V_{in}}(s) = \frac{1}{R_G\left\{-sC_{GD} + \left(sC_{in} + sC_{GD} + \frac{1}{R_G}\right)\frac{s(C_{GD}+C_{out})+\frac{1}{R_L}}{sC_{GD}-g_m}\right\}}$$

$$= \frac{R_L(sC_{GD}-g_m)}{R_G\left\{-R_L(sC_{GD}-g_m)sC_{GD} + \left\{s(C_{in}+C_{GD})+\frac{1}{R_G}\right\}\left(R_L s(C_{GD}+C_{out})+1\right)\right\}}$$

$$= \frac{R_L(sC_{GD}-g_m)}{R_L R_G(C_{GD}C_{out}+C_{in}C_{out}+C_{in}C_{GD})s^2 + (g_m R_L R_G C_{GD} + R_G C_{in} + R_G C_{GD} + R_L C_{GD} + R_L C_{out})s + 1}$$

Poles & Zeros

$$= \frac{R_L(sC_{GD} - g_m)}{R_L R_G(C_{GD}C_{out} + C_{in}C_{out} + C_{in}C_{GD})s^2 + (g_m R_L R_G C_{GD} + R_G C_{in} + R_G C_{GD} + R_L C_{GD} + R_L C_{out})s + 1}$$

- Zero:

$$\omega_z = \frac{g_m}{C_{XY}}.$$

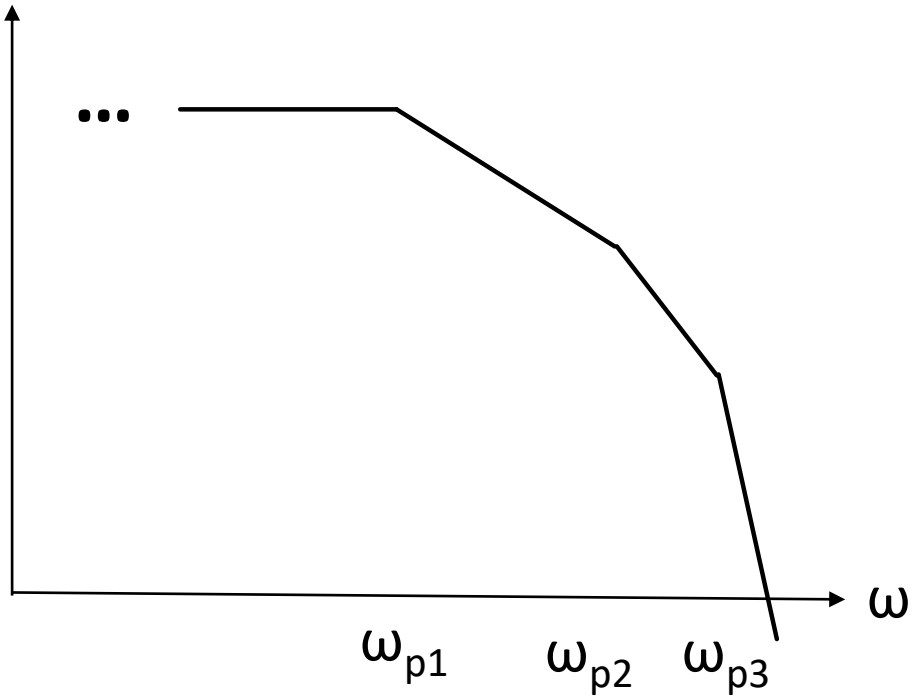
- Poles:

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)$$

$$= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

Dominant Pole in High Frequency

- Smallest poles among the high frequency poles “dominantly” determines the high frequency response of the circuit.
- Especially, when ω_{p1} << other poles



Back To CS Stage
High Frequency Response

- In the denominator,

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1.$$

- When $\omega_{p1} << \omega_{p2}$,

$$b = \frac{1}{\omega_{p1}}$$

- From the below CS stage Gain,

$$\frac{R_L(sC_{GD} - g_m)}{R_L R_G(C_{GD}C_{out} + C_{in}C_{out} + C_{in}C_{GD})s^2 + (g_m R_L R_G C_{GD} + R_G C_{in} + R_G C_{GD} + R_L C_{GD} + R_L C_{out})s + 1}$$

- We can say

$$\omega_{p1} = \frac{1}{(1 + g_m R_L)R_G C_{GD} + R_G C_{in} + R_L(C_{GD} + C_{out})}$$

Comparison w/ Miller Approximation

$$\omega_{p,in} = \frac{1}{(1+g_m R_L)R_G C_{GD} + R_G C_{in}}$$ (Pole from Miller Approximation)

$$\omega_{p1} = \frac{1}{(1+g_m R_L)R_G C_{GD} + R_G C_{in} + R_L (C_{GD} + C_{out})}$$ (Dominant Pole)

Non-dominant Pole

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1.$$

$$\frac{R_L(sC_{GD} - g_m)}{R_L R_G(C_{GD}C_{out} + C_{in}C_{out} + C_{in}C_{GD})s^2 + (g_m R_L R_G C_{GD} + R_G C_{in} + R_G C_{GD} + R_L C_{GD} + R_L C_{out})s + 1}$$

- Non-dominant pole is determined by

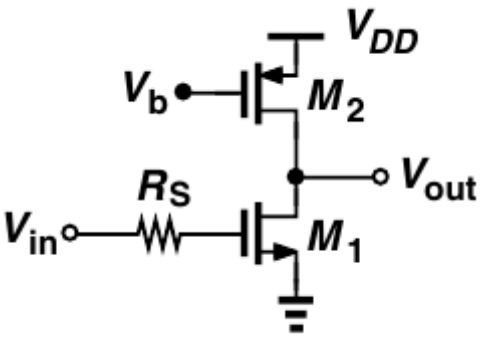
$$\omega_{p2} = b / a$$

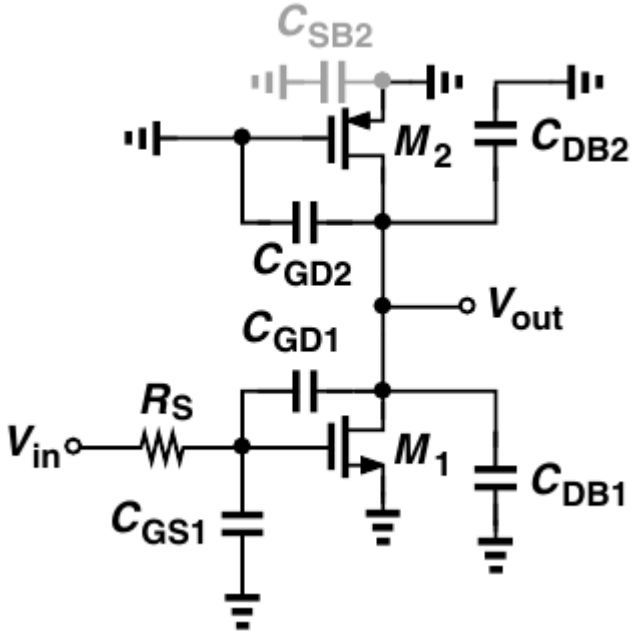
- That is,

$$\omega_{p2} = \frac{(1 + g_m R_L)R_G C_{GD} + R_G C_{in} + R_L(C_{GD} + C_{out})}{R_L R_G(C_{GD}C_{out} + C_{in}C_{out} + C_{in}C_{GD})}$$

Example 11.17

Using the dominant-pole approximation, compute the poles of the circuit shown in Fig. 11.31(a). Assume both transistors operate in saturation and $\lambda \neq 0$.





$$\omega_{p1} = \frac{1}{(1+g_m R_L)R_G C_{GD} + R_G C_{in} + R_L(C_{GD} + C_{out})}$$

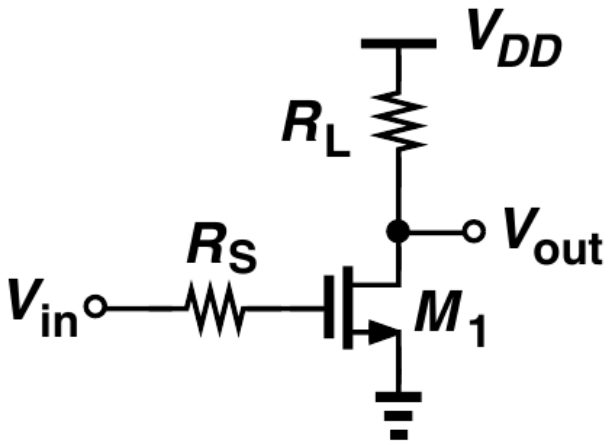
$$\omega_{p2} = \frac{(1+g_m R_L)R_G C_{GD} + R_G C_{in} + R_L(C_{GD} + C_{out})}{R_L R_G (C_{GD}C_{out} + C_{in}C_{out} + C_{in}C_{GD})}$$

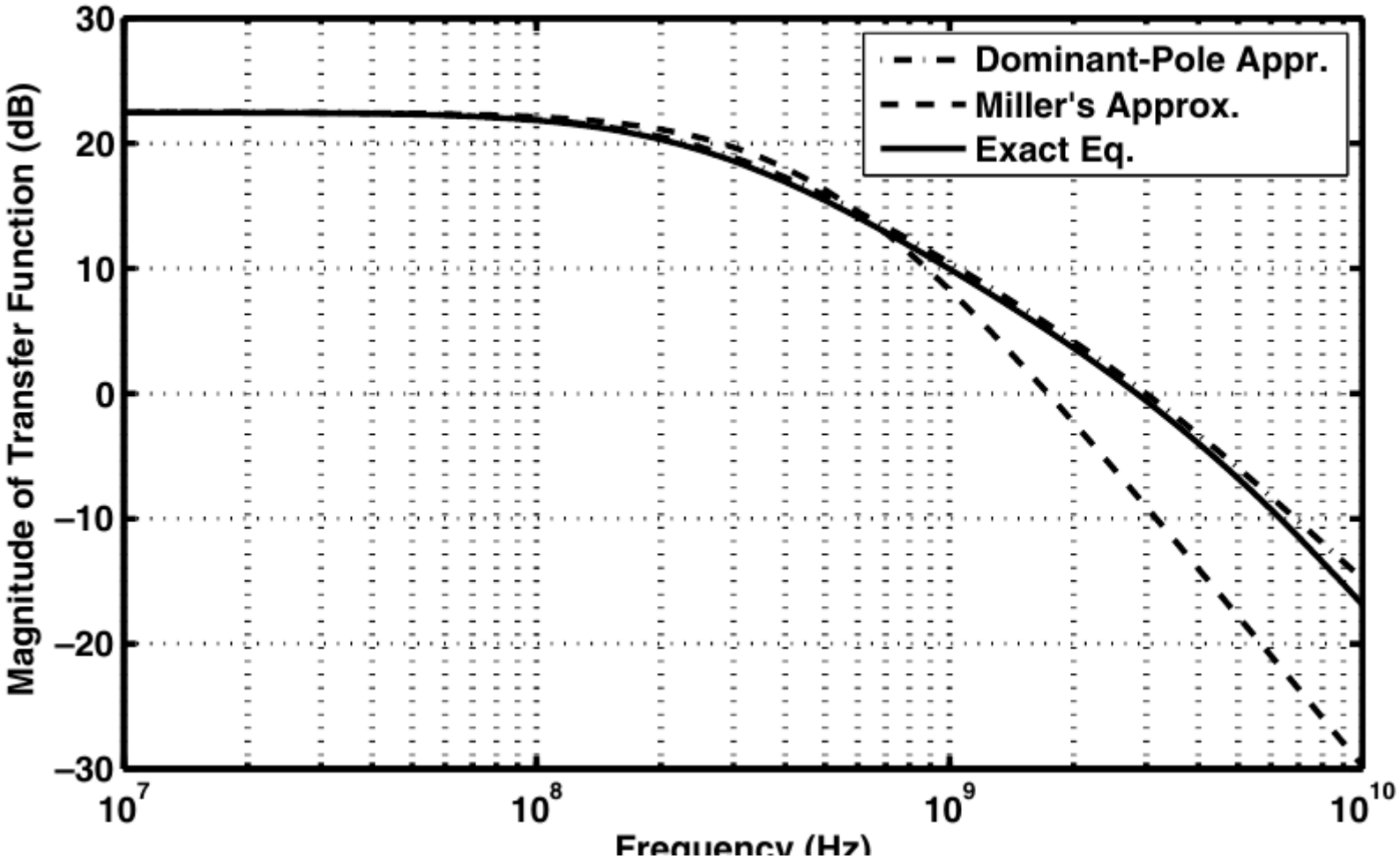
Example 11.18

In the CS stage of Fig. 11.29(a), we have $R_S = 200\,\Omega, C_{GS} = 250$ fF, $C_{GD} = 80$ fF, $C_{DB} = 100$ fF, $g_m = (150\,\Omega)^{-1}$, $\lambda = 0$, and $R_L = 2\,\text{k}\Omega$. Plot the frequency response with the aid of (a) Miller's approximation, (b) the exact transfer function, (c) the dominant-pole approximation.

$$\omega_{p,in} = \frac{1}{R_G[C_{in} + (1 + g_m R_L)C_{GD}]}$$

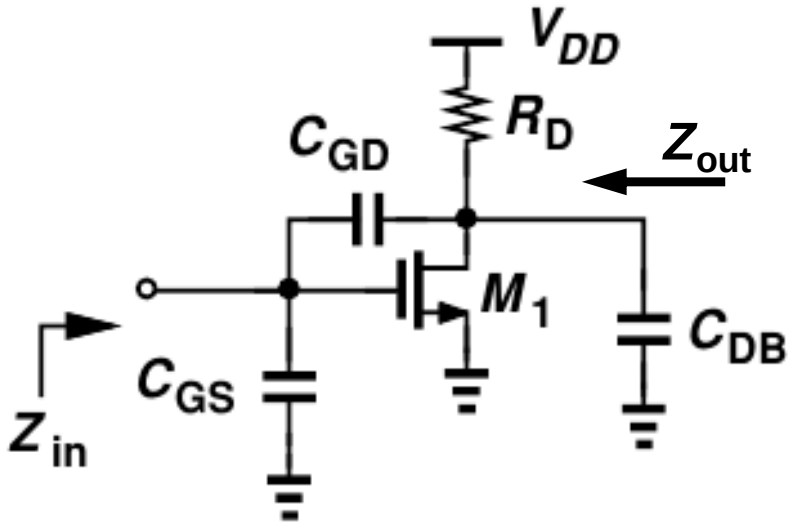
$$\omega_{p,out} = \frac{1}{R_L[C_{out} + (1 + \frac{1}{g_m R_L})C_{GD}]}$$




Dominant-Pole Appr.
Miller's Approx.
Exact Eq.
Magnitude of Transfer Function (dB)
30
20
10
0
−10
−20
−30
10^7
10^8
10^9
10^{10}
Frequency (Hz)

Input/Output Impedance

- With a high voltage gain, the Miller effect may substantially lower the input impedance at high frequencies.



$$Z_{in} \approx \frac{1}{[C_{GS} + (1 + g_m R_D) C_{GD}]s}$$

$$Z_{out} \approx R_D \parallel \frac{1}{s\left\{C_{DB} + \left(1 + \frac{1}{g_m R_D}\right) C_{GD}\right\}}$$

Summary

- The frequency response of CS stage can be analyzed in two aspects; low frequency and high frequency roll off
- Low frequency roll-off is determined by Ci and Cb
- High frequency roll-off is determined by transistor caps.
- Dominant pole approach can show good accuracy when there is a significant difference between two poles.