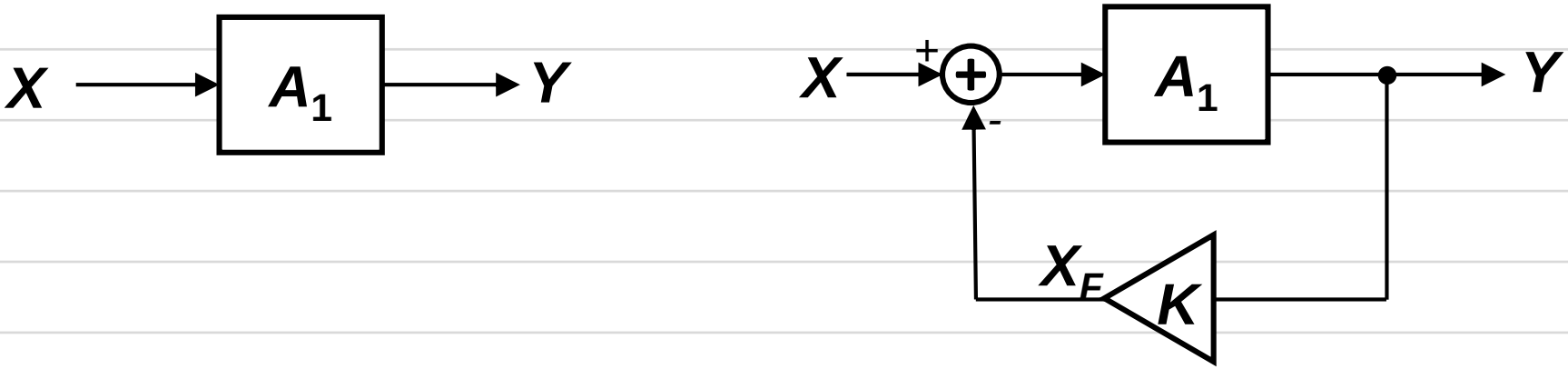
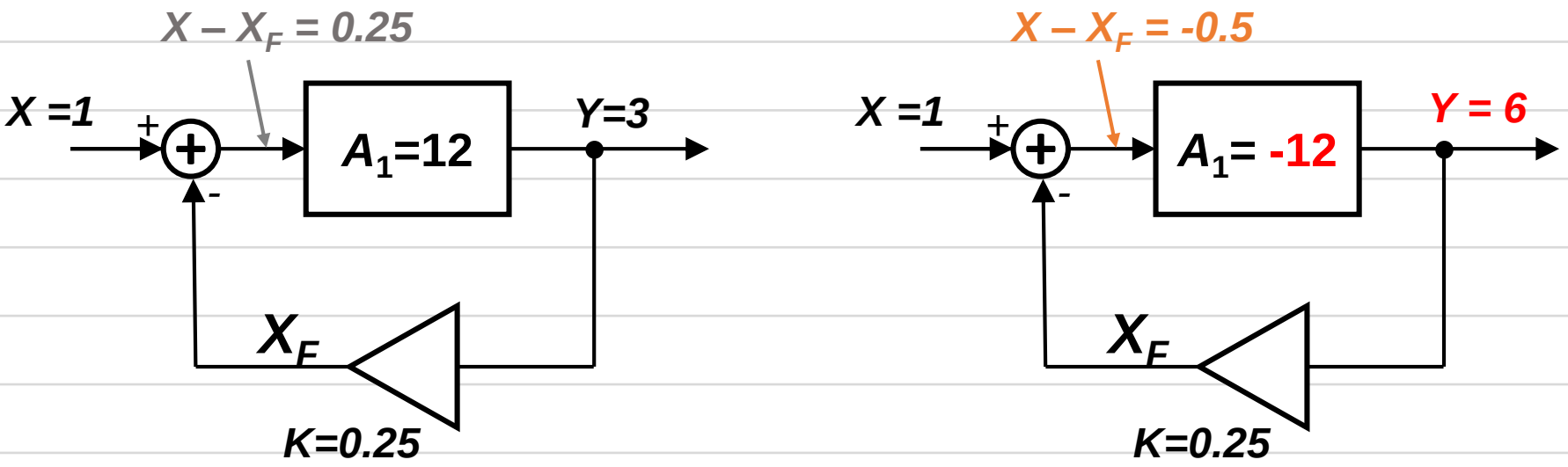
Output Determination in Feedback

- $Y = A_1(X-X_F) = A_1(X-KY)$ ➔ $(1+KA_1)Y = AX$ ➔ $Y/X = \frac{A}{1+KA_1}$



Divergence in Feedback System

- Negative vs. Positive feedback



Stability in Feedback

Hanwool Jeong

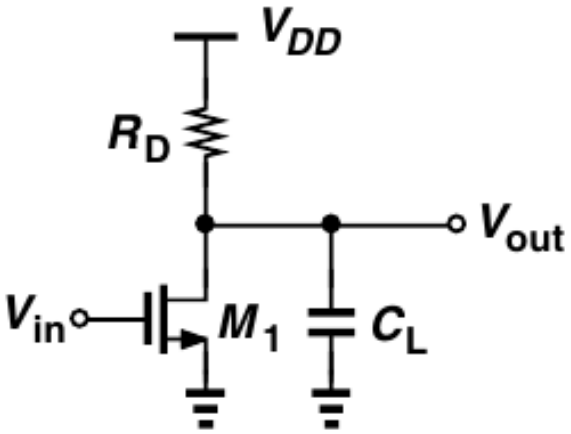
hwjeong@kw.ac.kr

Review of Magnitude Bode Plot

- Construct the Bode plot of |H(jω)|

$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m\left(R_D \| \frac{1}{C_L s}\right) = \frac{-g_m R_D}{R_D C_L s + 1}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}.$$



- Dividing ω into two region,

Generalizing Bode Rule in Magnitude Plot

|H(jω)| can be approximated with the following rules:

- ω passes each pole frequency: |H(jω)| slope decreases by 20dB/dec
- ω passes each zero frequency: |H(jω)| slope increases by 20 dB/dec.

$$H(s) = H_0 \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\left(1+\frac{s}{z_3}\right)\cdots}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\cdots} \Rightarrow |H(j\omega)| = |H_0| \frac{\sqrt{1+\frac{\omega^2}{z_1^2}}\sqrt{1+\frac{\omega^2}{z_2^2}}\sqrt{1+\frac{\omega^2}{z_3^2}}\cdots}{\sqrt{1+\frac{\omega^2}{p_1^2}}\sqrt{1+\frac{\omega^2}{p_2^2}}\sqrt{1+\frac{\omega^2}{p_3^2}}\cdots}$$

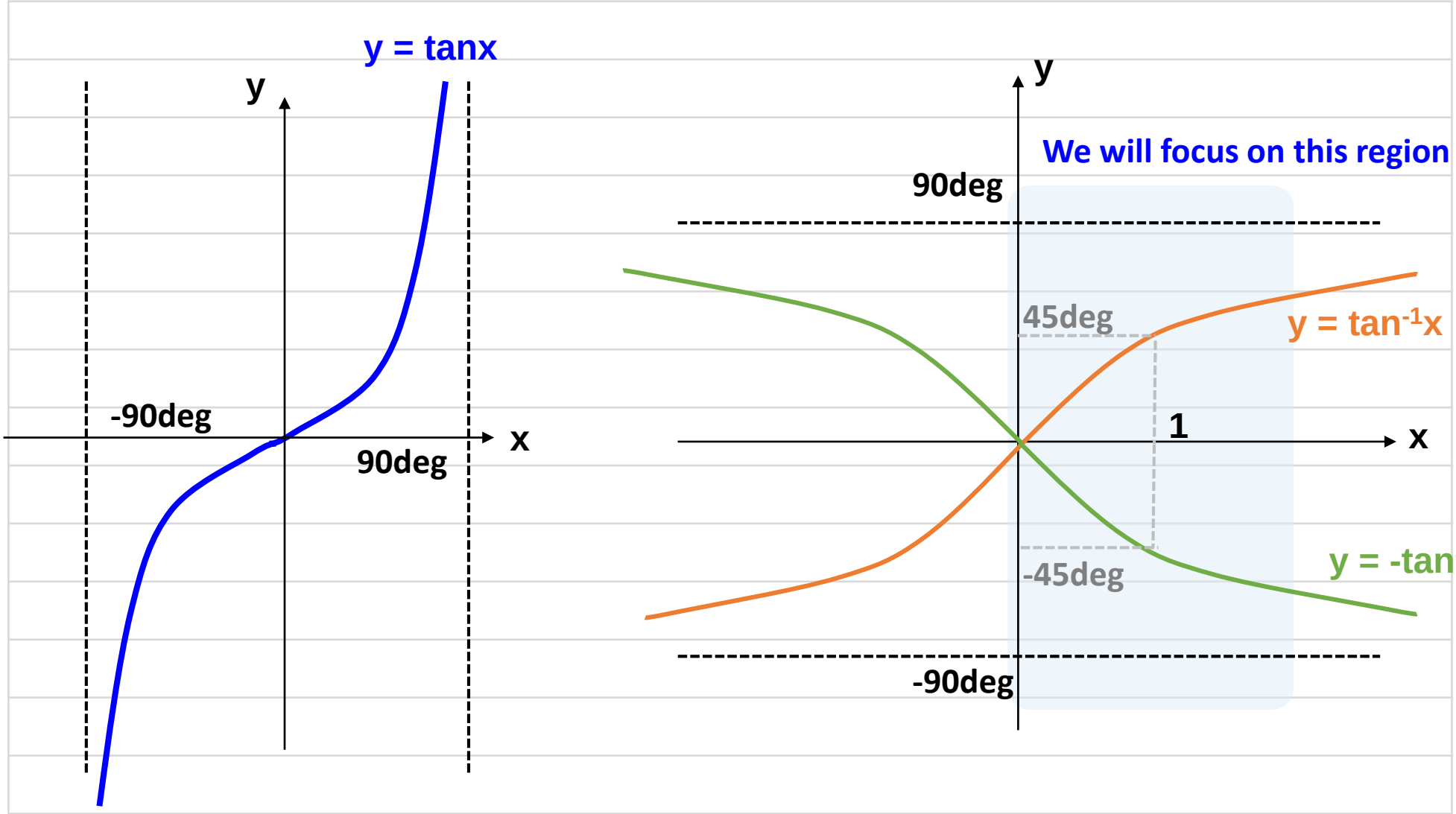
How About Phase?

- Can you draw phase plot for

$$H(s) = H_0 \frac{\left(1+\frac{s}{z_1}\right)}{\left(1+\frac{s}{p_1}\right)} \Rightarrow H(j\omega) = H_0 \frac{\left(1+\frac{j\omega}{z_1}\right)}{\left(1+\frac{j\omega}{p_1}\right)} \Rightarrow \angle H(j\omega) = \angle H_0 + \tan^{-1}\left(\frac{\omega}{z_1}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right)$$

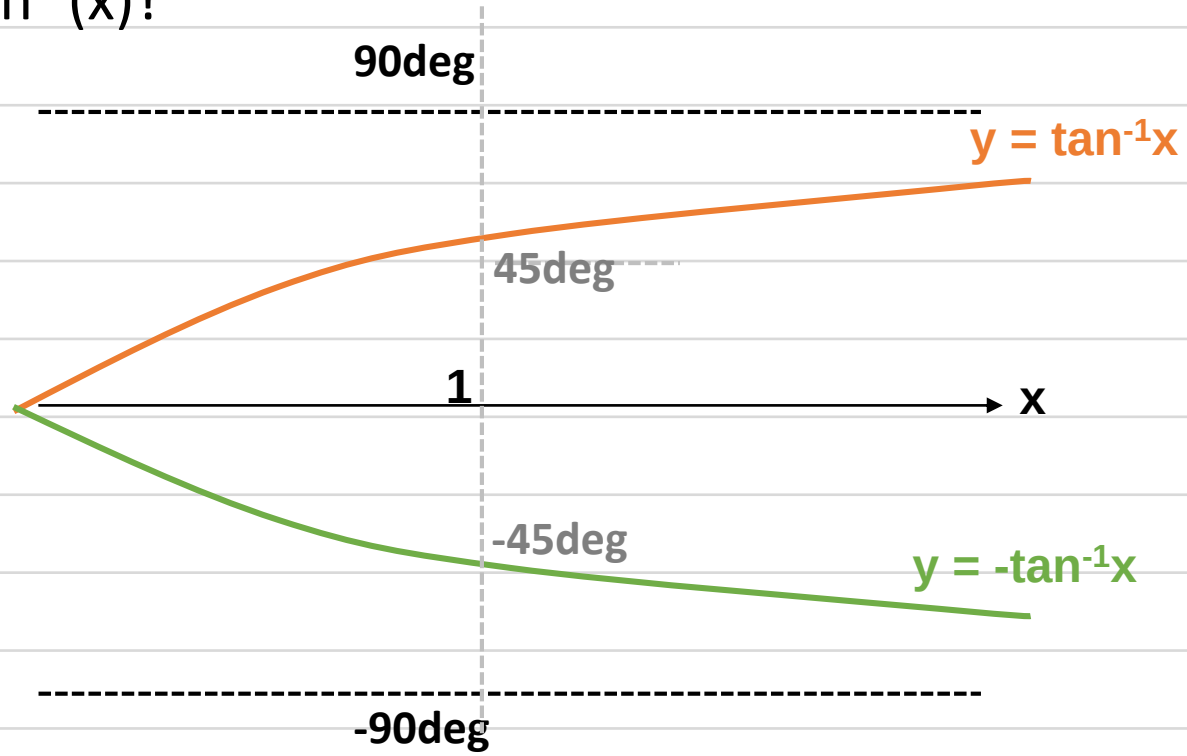
- How can you handle $\tan^{-1}(\omega/z)$ or $-\tan^{-1}(\omega/p)$?

Graph for $y = \tan^{-1}x$



Graph for $y = \pm \tan^{-1}x$ Near x=1

- We approximate the $\tan^{-1}(x)$ graph as,
  1) When $x << 1$ or $x = 0.1$, $\tan^{-1}x$ start to increase from near 0 deg
  2) When x =1, tan-1x becomes 45 deg
  3) When $x>>1$ or $x = 10$, $\tan^{-1}x$ stop to increase near 90 deg
- How about $-\tan^{-1}(x)$?



How About Graph for $\pm y = \tan^{-1}(x/k)$ Near x=k?

- We can approximate the $\tan^{-1}(x/k)$ graph as,
  1) When x << k or x = 0.1kk $\tan^{-1}x$ start to increase from near 0 deg.
  2) When x =k, $\tan^{-1}x$ becomes 45 deg.
  3) When x>>k or x = 10k, $\tan^{-1}x$ stop to increase near 90 deg.

Can You Generalize it and Establish Bode Rule for Phase Plot?

- Can you draw phase plot for

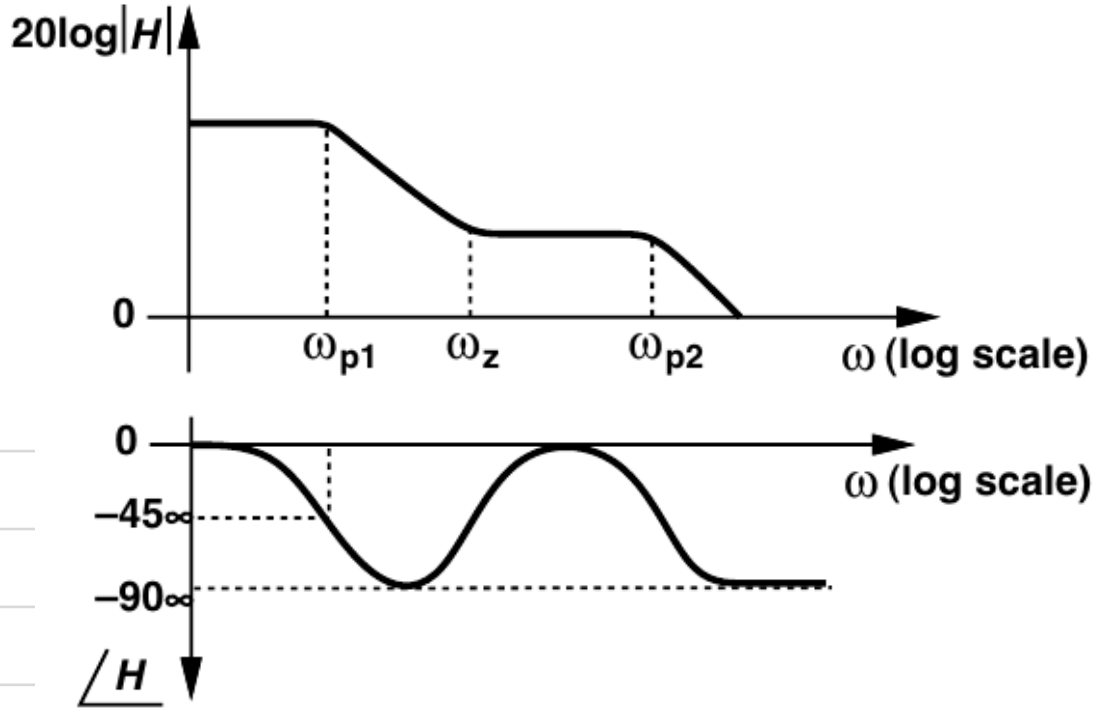
$$H(s) = H_0 \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\left(1+\frac{s}{z_3}\right)\cdots}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\cdots} \Rightarrow \angle H(j\omega) = \angle\left[H_0 \frac{\left(1+\frac{j\omega}{z_1}\right)\left(1+\frac{j\omega}{z_2}\right)\left(1+\frac{j\omega}{z_3}\right)\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)\left(1+\frac{j\omega}{p_3}\right)\cdots}\right]$$

- For zero, the phase of a transfer function
  1) Begins to **increase** at ω=1/10z
  2) Incurs a change of **+45 deg** at ω=z
  3) Experiences a total change of **+90 deg** nearly at ω=10z.
- For pole, the phase of a transfer function
  1) Begins to **decrease** at ω=1/10p
  2) Incurs a change of **-45 deg** at ω=p
  3) Experiences a total change of **-90 deg** nearly at at ω=10p.

Example 12.34

Figure 12.61(a) depicts the magnitude response of an amplifier. Using Bode's rule, plot the phase response.[15]

- Assume that DC gain H_0 is positive



- In this example, we have assumed that the pole and zero frequencies are so wide apart that 10ω p1 < 0.1ω z and 10ω z < 0.1ω p2 . In practice, this may not hold, requiring more detailed calculations.
- **How would it be different if H_0 is negative?**

How About “s” Term?

- How can we handle the following transfer function?

$$H(s) = H_0 \frac{\boldsymbol{s}\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\left(1+\frac{s}{z_3}\right)\cdots}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\cdots}$$

Zero at origin

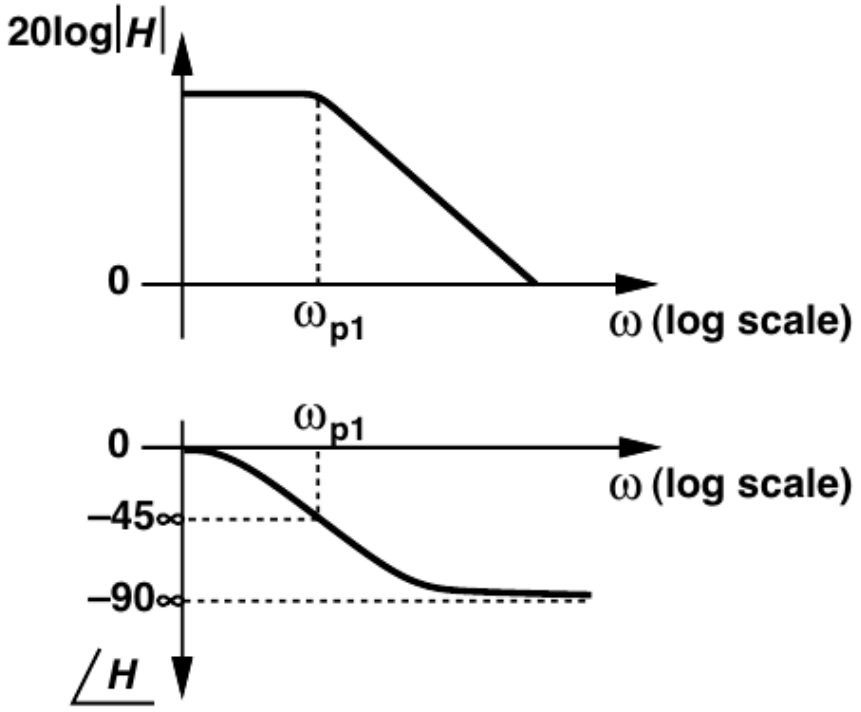
$$H(s) = H_0 \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\left(1+\frac{s}{z_3}\right)\cdots}{\boldsymbol{s}\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\cdots}$$

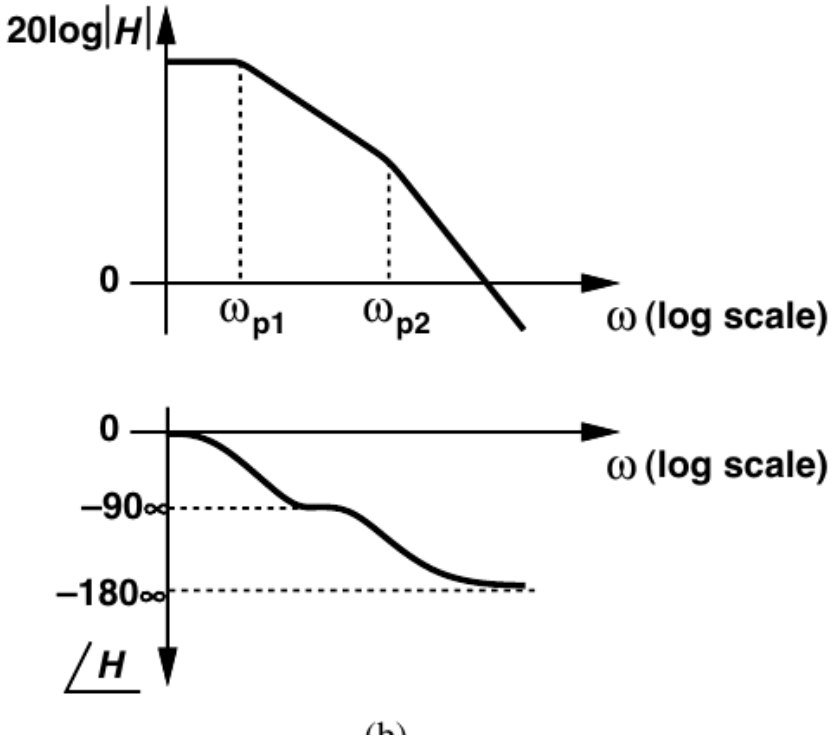
Pole at origin

- They would include additionally $\tan^{-1}(\text{inf})$ or $-\tan^{-1}(\text{inf})$, which is correspond to +90 deg or -90 deg, respectively.

Example 12.35

Construct the magnitude and phase response of an amplifier having (a) one pole, (b) two poles, or (c) three poles.

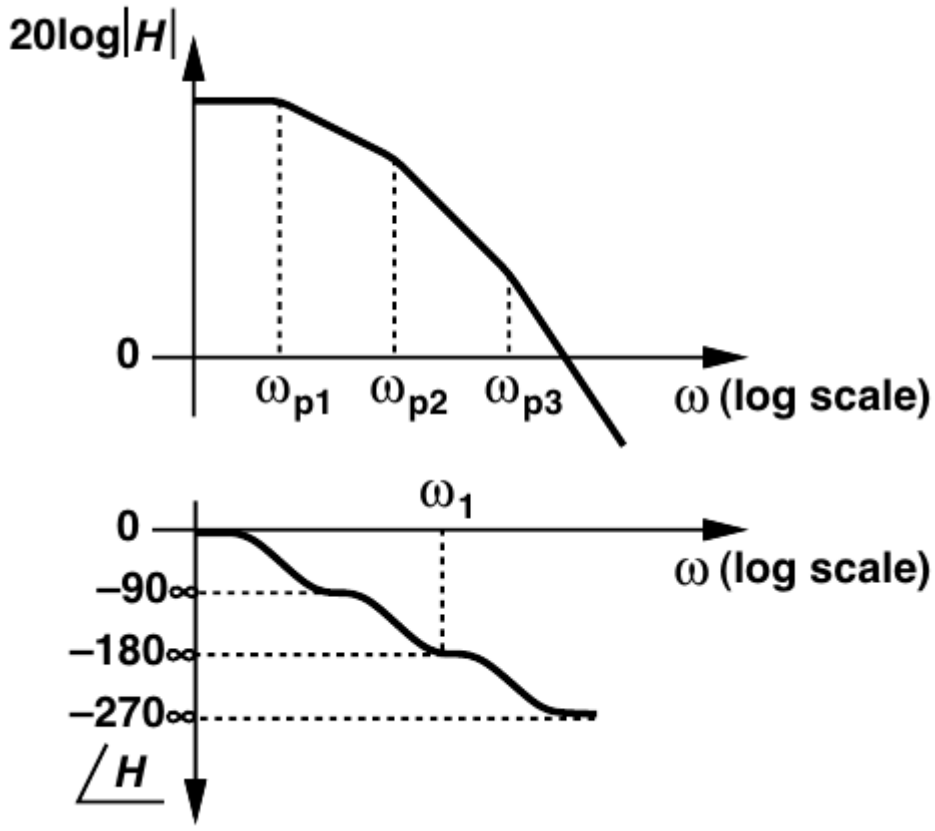


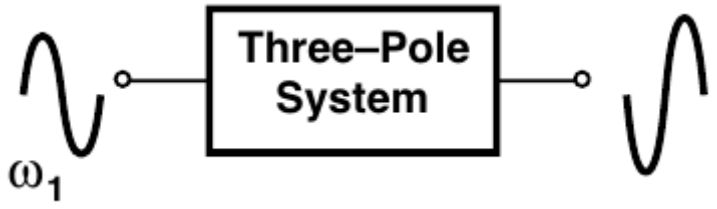


(b)

Example 12.35

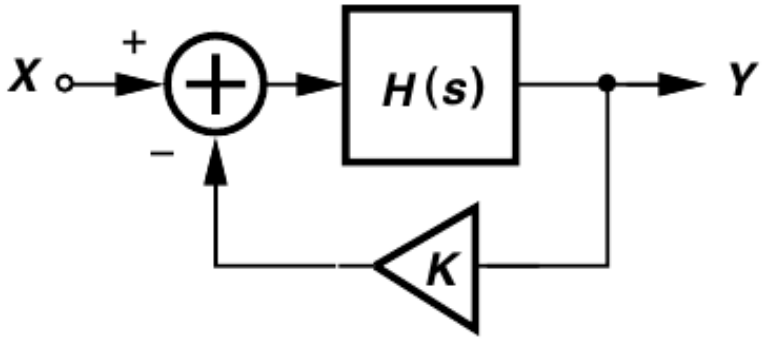
Construct the magnitude and phase response of an amplifier having (a) one pole, (b) two poles, or (c) three poles.





Problem of Instability

- **Question is what if "KH = -1" at a certain ω?**



$$\frac{Y}{X}(s) = \frac{H(s)}{1 + KH(s)}$$

- That is, what if

$$|KH(j\omega_1| = 1 \qquad \angle KH(j\omega_1) = -180^\circ$$

- **We say that the system oscillates or is unstable at ω_1**
- **More generally,**

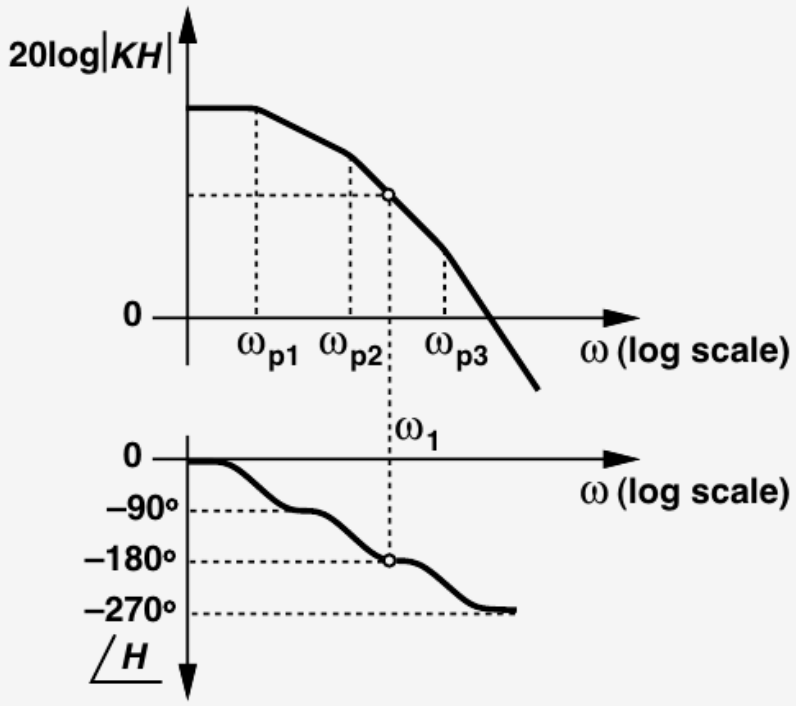
$$|KH(j\omega_1| > 1 \qquad \angle KH(j\omega_1) = -180^\circ$$

Example 12.36

Explain why a two-pole system cannot oscillate

Example 12.37

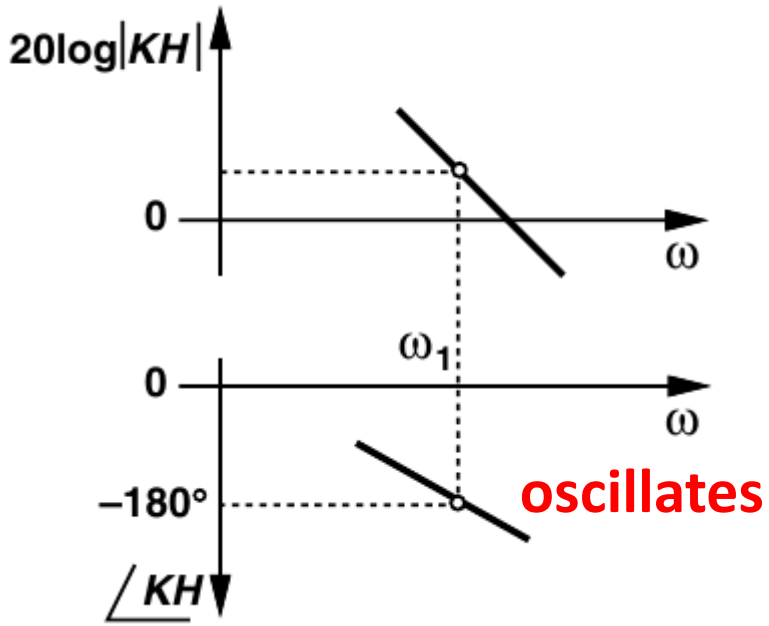
A three-pole feedback system exhibits the frequency response depicted in Fig. 12.65. Does this system oscillate?



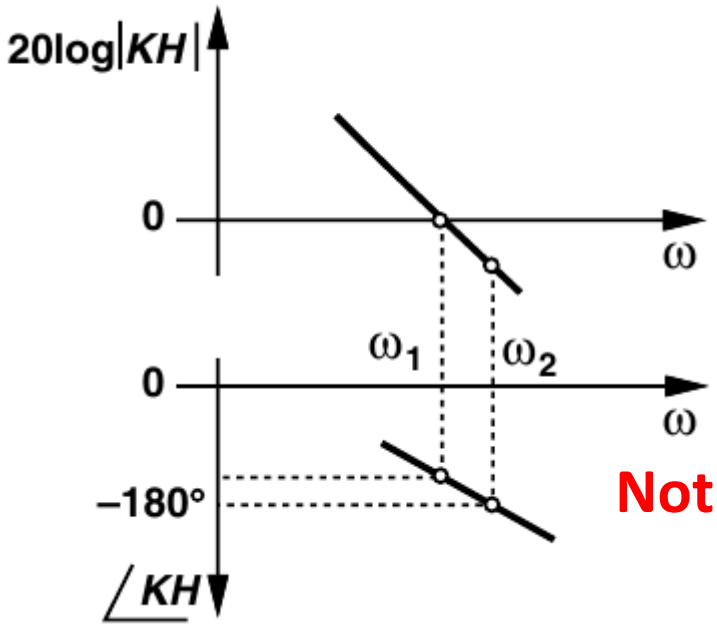
Stability Condition

- To avoid instability, we must ensure that these two conditions do not occur at the same frequency.

$$|KH(j\omega_1)| \geq 1 \text{ and } \angle H(j\omega_1) = -180^\circ$$



oscillates



Not oscillates

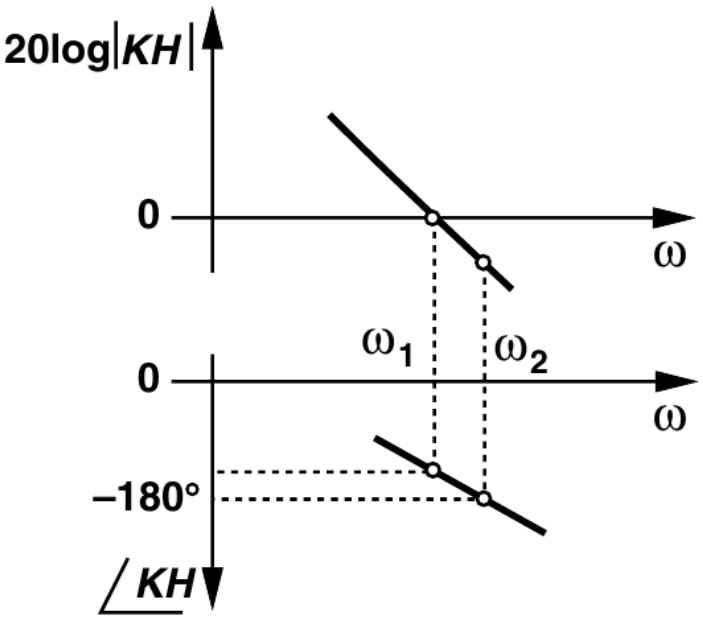
Gain Crossover Frequency & Phase Crossover Frequency

- Gain crossover frequency (ω_{GX})
  : Frequency when **the loop gain** is unity

- Phase crossover frequency (ω_{PX})
  : Frequency when phase shift is -180deg

- Stability requires

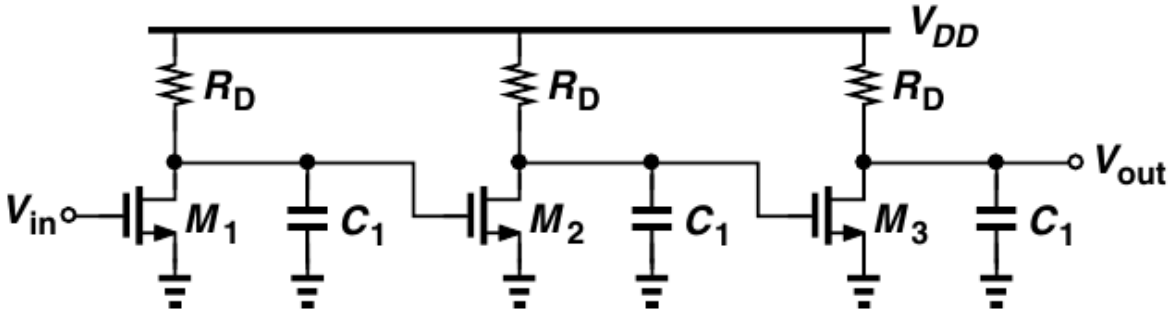
$$\omega_{GX} < \omega_{PX}$$

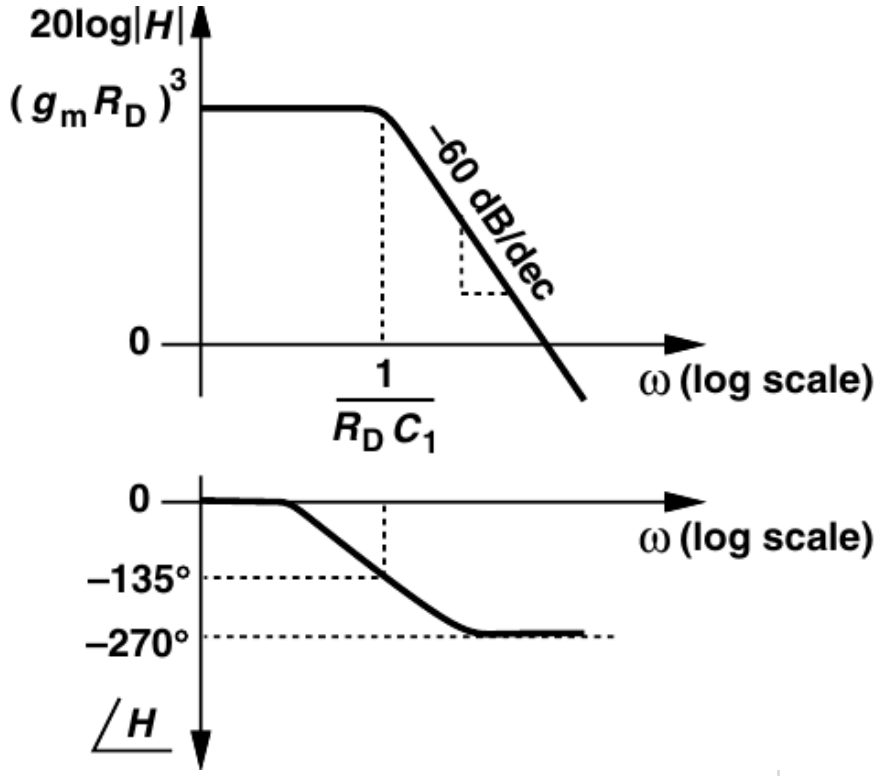


- To guarantee stability in negative-feedback systems, we must ensure that the loop gain falls to unity before the phase shift reaches −180◦

Example 12.38

We wish to apply negative feedback with $K = 1$ around the three-stage amplifier shown in Fig. 12.67(a). Neglecting other capacitances and assuming identical stages, plot the frequency response of the circuit and determine the condition for stability. Assume $\lambda = 0$.



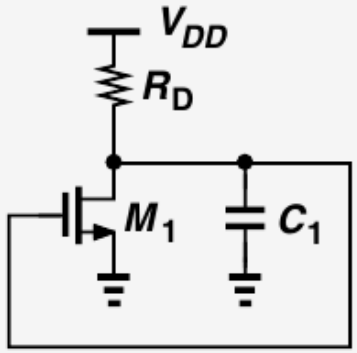


$$H(s) = \frac{(g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3}, \qquad \angle H(j\omega) = -3\tan^{-1}\frac{\omega}{\omega_p}$$

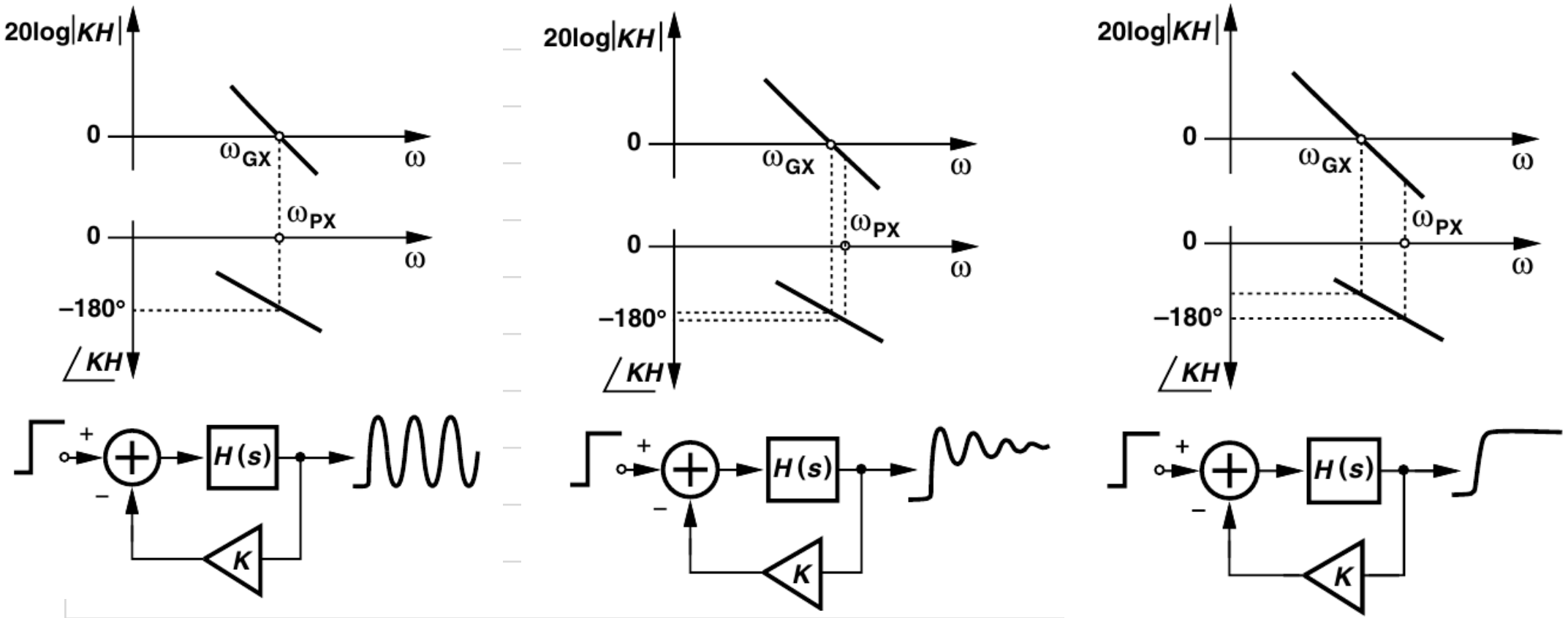
$$\frac{(g_m R_D)^3}{\left[\sqrt{1 + \left(\frac{\omega_{PX}}{\omega_p}\right)^2}\right]^3} < 1.$$

Example 12.39

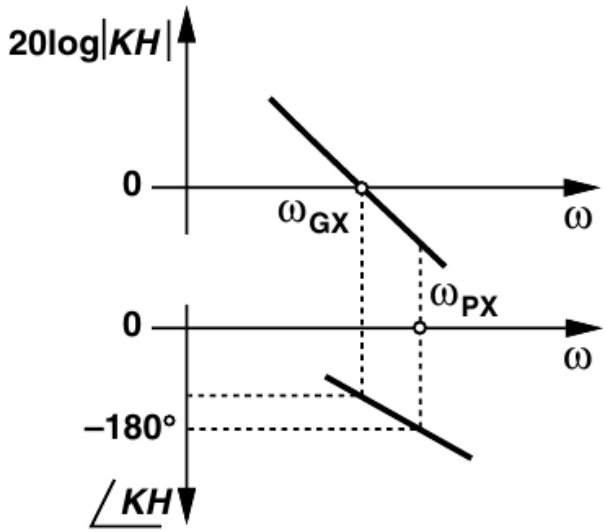
A common-source stage is placed in a unity-gain feedback loop as shown in Fig. 12.68. Explain why this circuit does not oscillate.



System Response According to $\omega_{GX}\sim\omega_{PX}$ or $\angle H(\omega_{GX})\sim -180^{o}$



Phase Margin

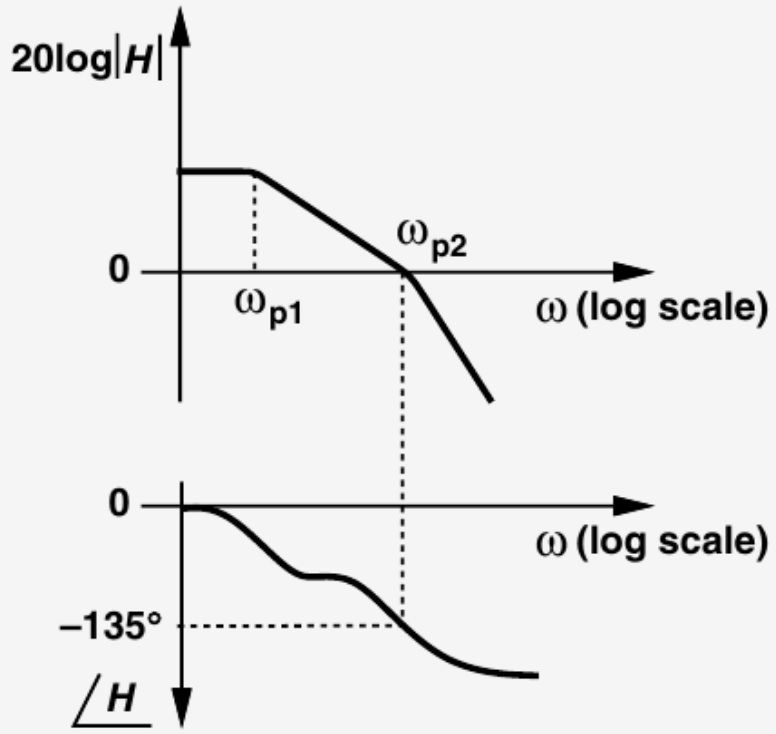


- A measure that tells how much feedback systems are stable is the **phase margin**.

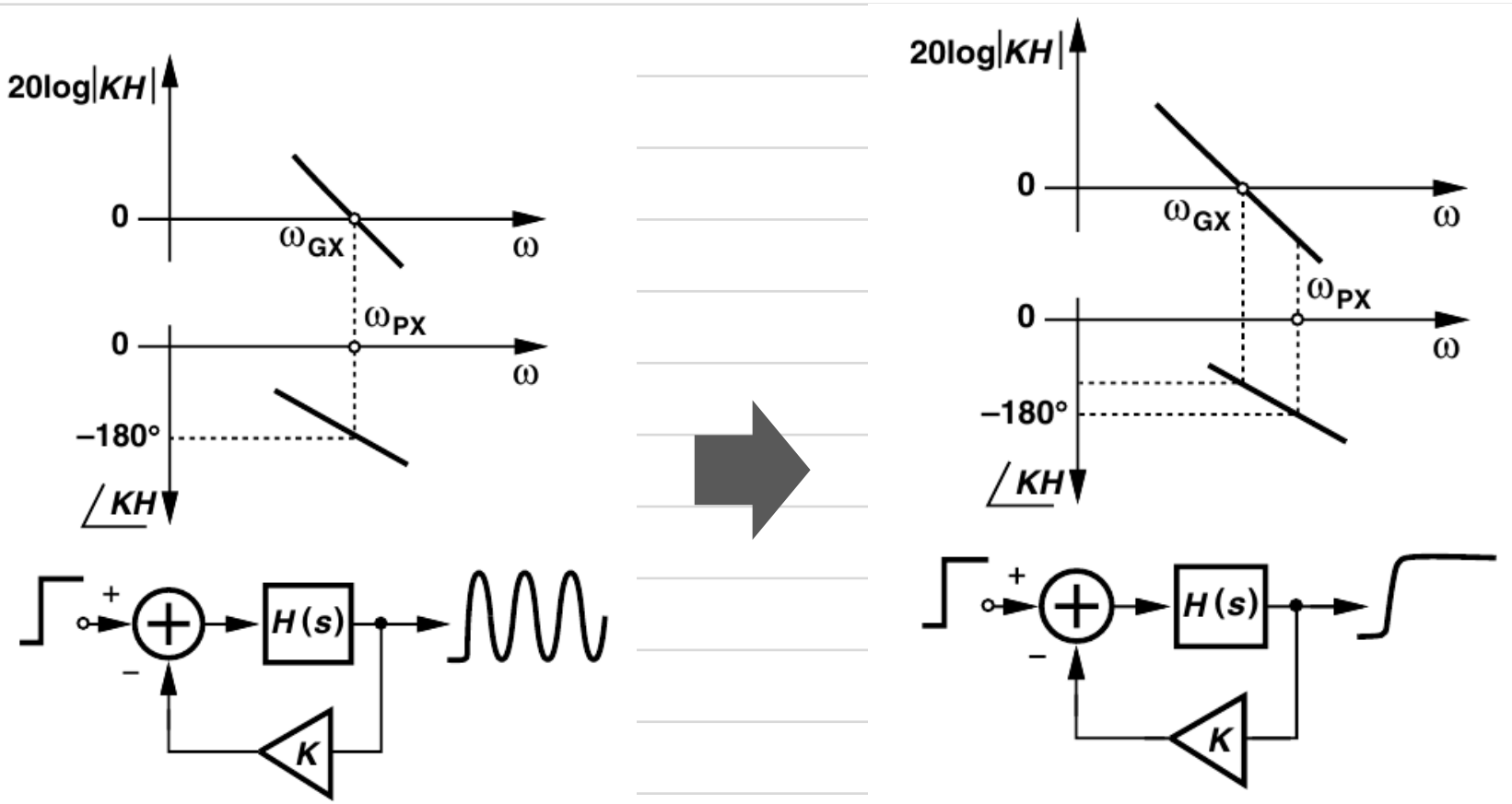
$$\text{Phase Margin} = \angle H(\omega_{GX}) + 180^\circ$$

Example 12.41

Figure 12.71 plots the frequency response of a multipole amplifier. The magnitude drops to unity at the second pole frequency. Determine the phase margin of a feedback system employing this amplifier with $K = 1$.



How Can We Make System Stable?

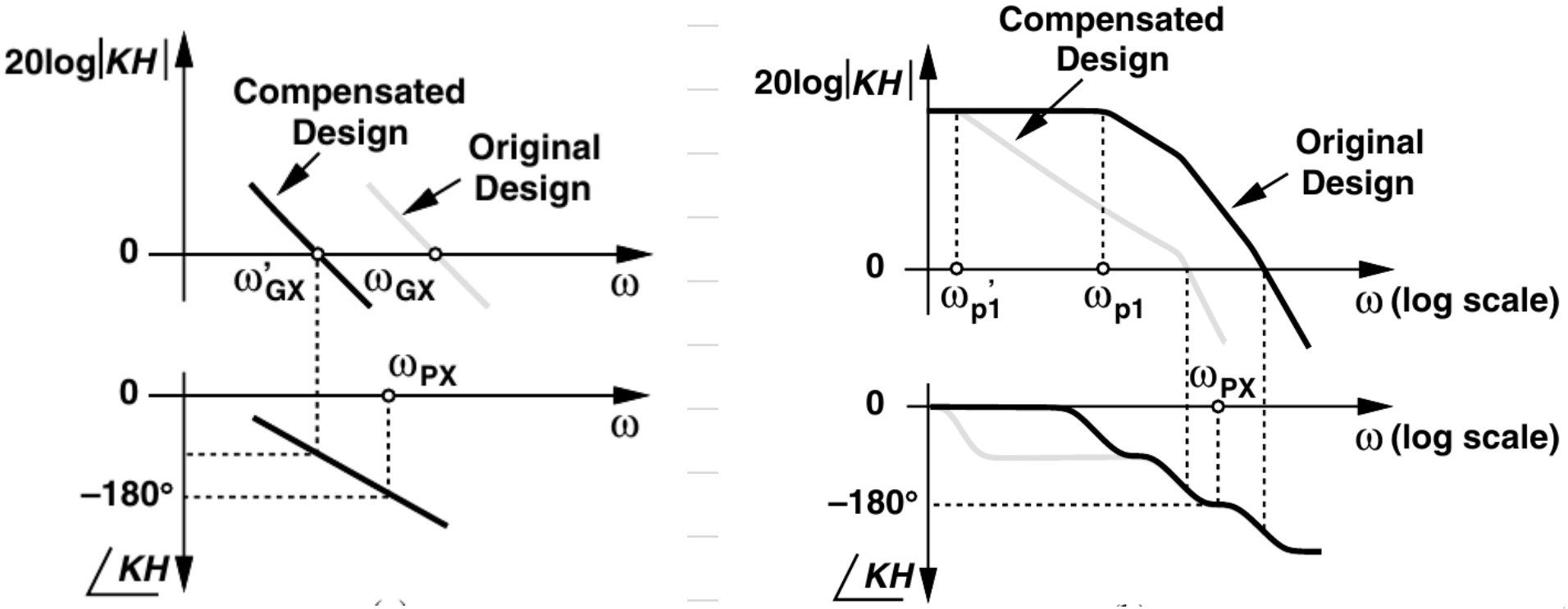


Frequency Compensation

- How?

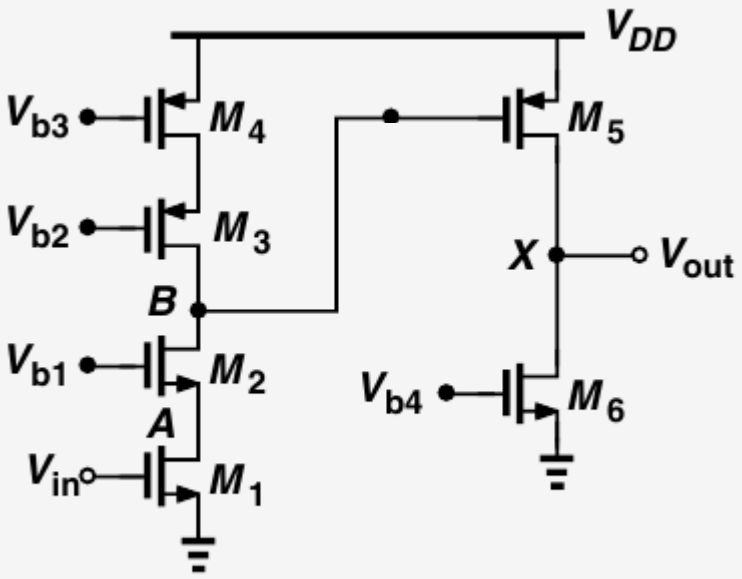
➔If the **dominant pole** is translated to lower frequencies, so is ω_{GX}.

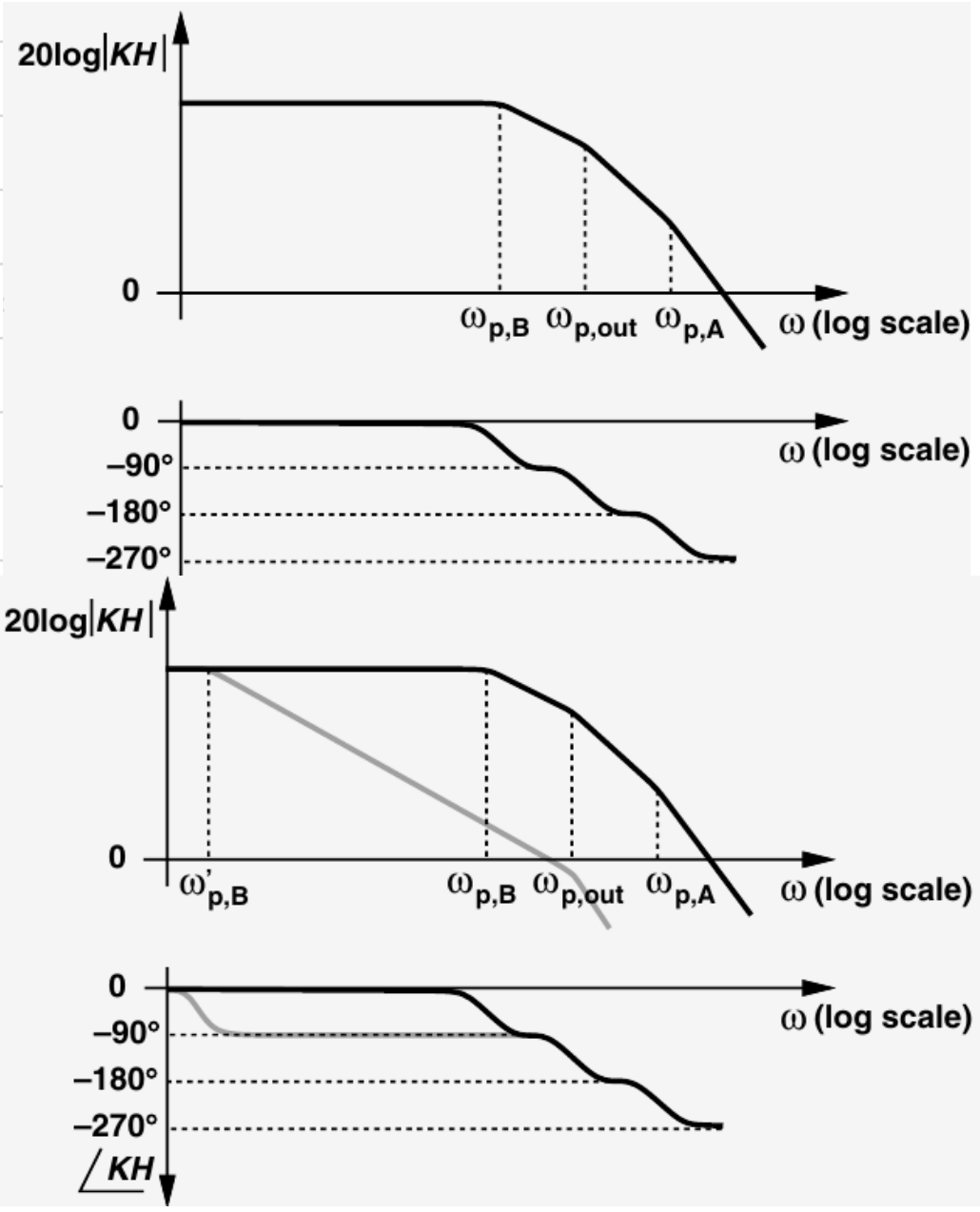
➔**Note that it sacrifices gain.**



Example 12.42

The amplifier shown in Fig. 12.73(a) employs a cascode stage and a CS stage. Assuming that the pole at node B is dominant, sketch the frequency response and explain how the circuit can be "compensated."

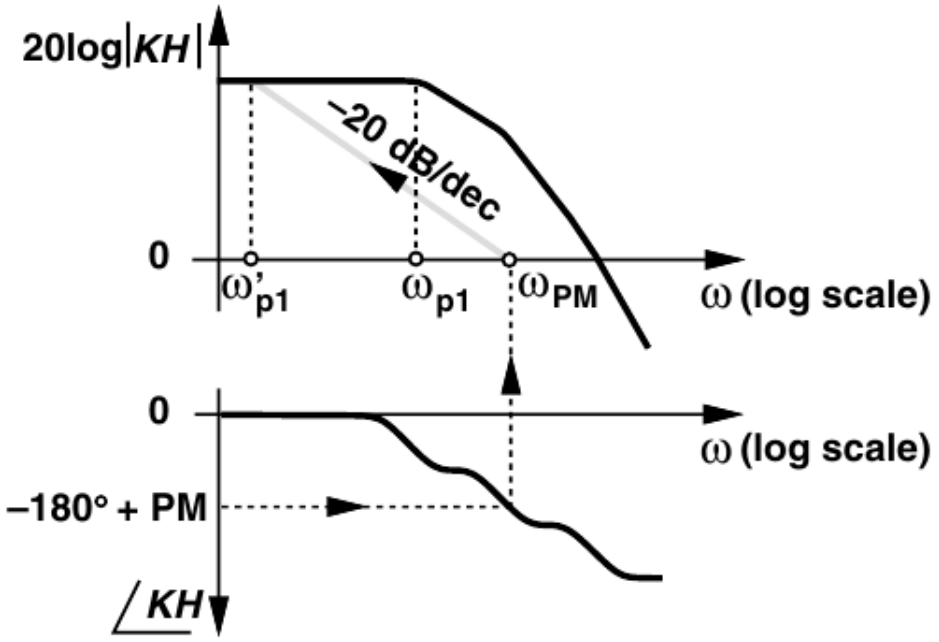




By How Much?

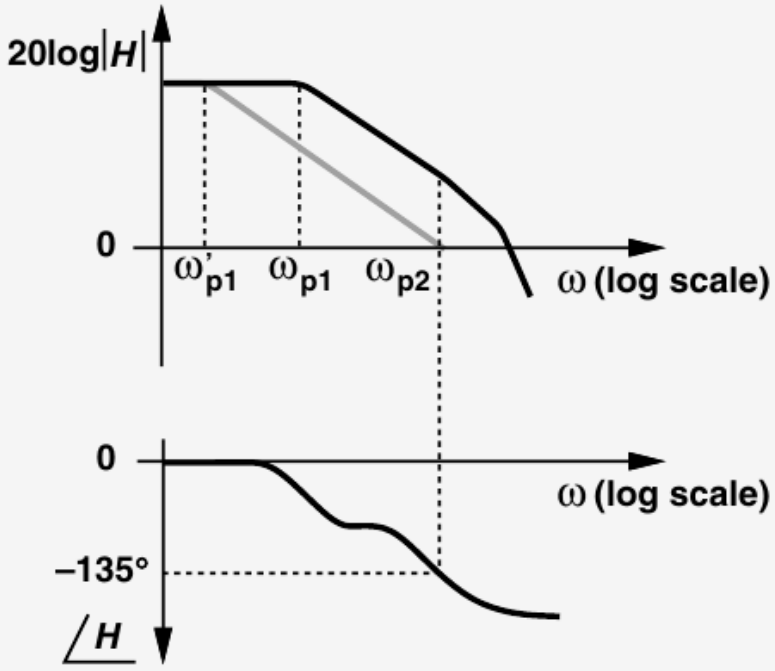
- Suppose that we are given with the target **PM**. Can you determine the target dominant pole ω'_{p1} ?

1) At the frequency of -180^o + PM, ω_{PM}, the gain should be 1.
2) That is, $|H(\omega_{PM})| = 0dB$
3) Then, with the line of the slope of -20dB/dec, you can find the ω'_{p1}

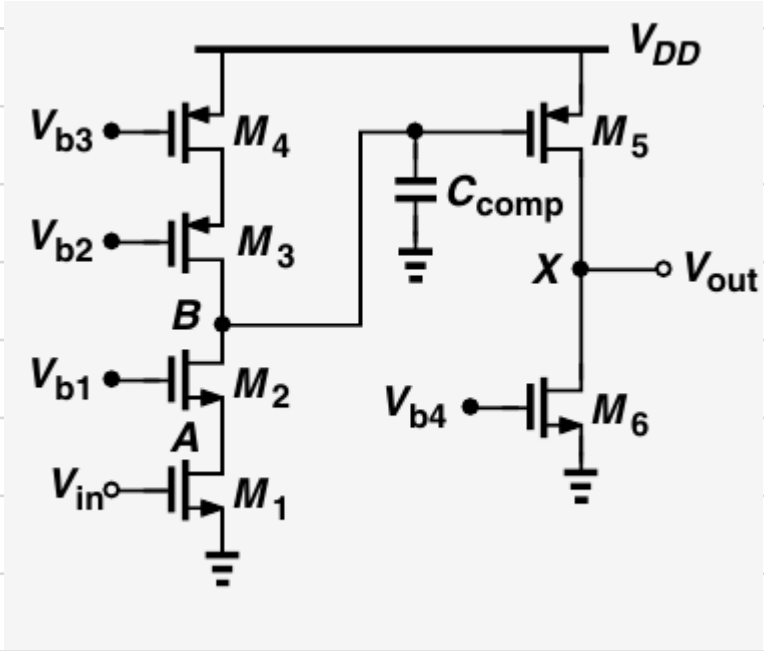


Example 12.43

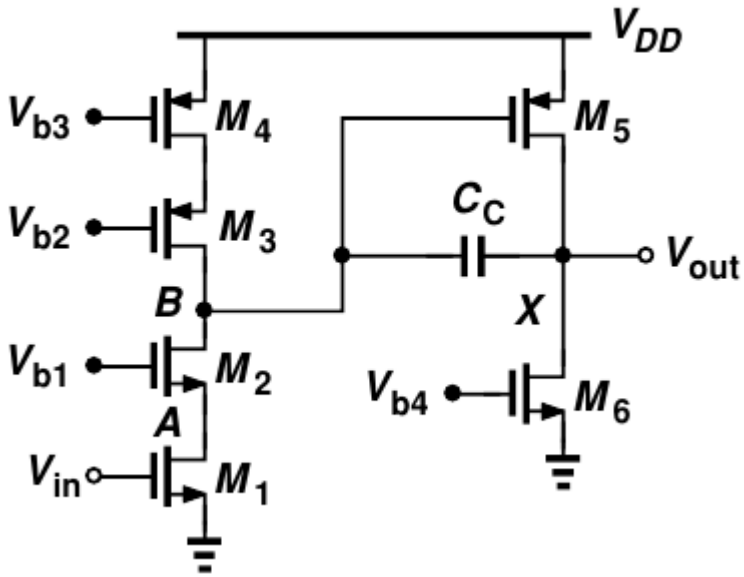
A multipole amplifier exhibits the frequency response plotted in Fig. 12.75(a). Assuming the poles are far apart, compensate the amplifier for a phase margin of 45° with $K = 1$.



Miller Compensation



vs.



Summary

- In the feedback system, the feedback polarity can change according to frequency
- Thus, the feedback can unexpectedly oscillate in a certain frequency although it forms negative feedback and stable otherwise.
- A measure that tells how much feedback systems are stable is the phase margin.
- To stabilize the feedback system, the dominant pole should be reduced through the circuit design revision, which means the phase margin is increased.