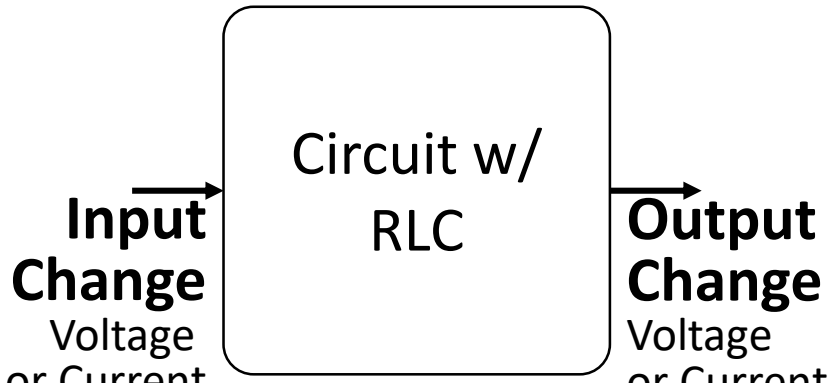
What We Learned & What We Will Learn + Additional Things Helpful

What We Learned

	I-V relation	Steady State
Resistor	$v = iR$	$v = iR$
Capacitor	$i = C\frac{dv}{dt}$	$i=0$ (open)
Inductor	$v = L\frac{di}{dt}$	$v=0$ (short)

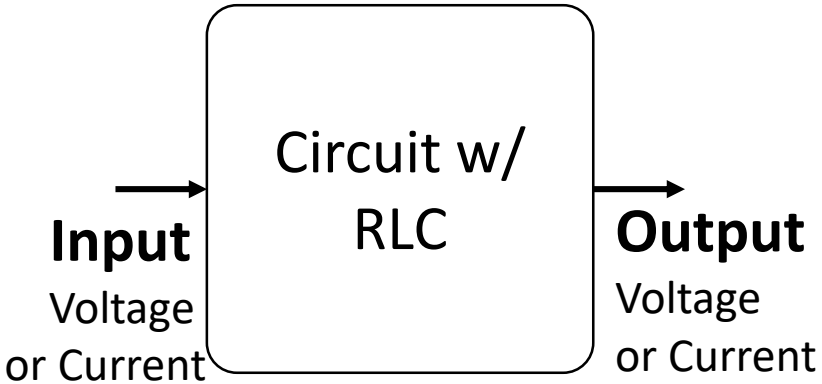
What We will Learn



"Response"

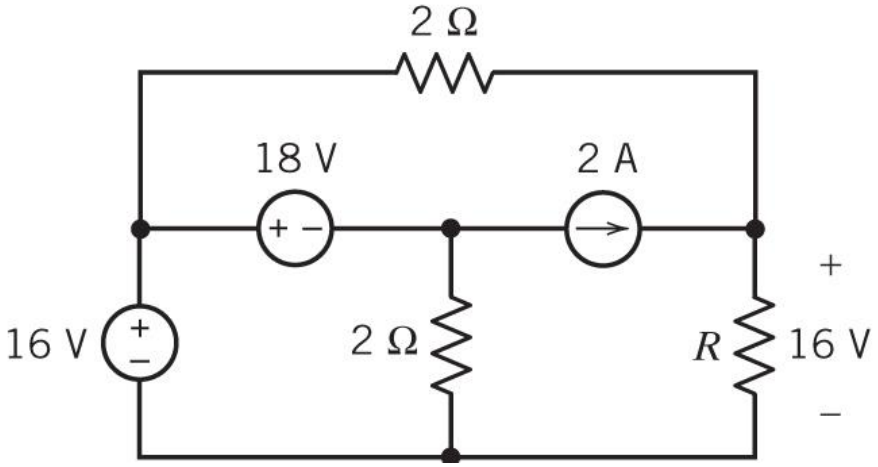
- ✓ **Voltage across a capacitor cannot change abruptly**
- ✓ **Current through an inductor cannot change abruptly**

What Do We Need for Deriving Response?



"Response"

We already did it for R only circuit as:



How?

➔ Repeatedly applying I-V relations of all components

➔ Combining linear equations

Why linear?

➔ Mainly because we use V=IR

What if in RLC?

- Each component of RLC circuit has differential relation in I-V.

➔ Instead of simple linear equation systems, **differential equations** for the given circuit are formed.

$$\frac{v_x - 3}{5} + \frac{8 - 3v_x}{4} = 2$$

R only

➡

1st order

$$3\frac{dv_x}{dt} + 5v_x = 2$$

2nd order

$$2\frac{d^2v_x}{dt^2} + 4\frac{dv_x}{dt} + 5v_x = 7$$

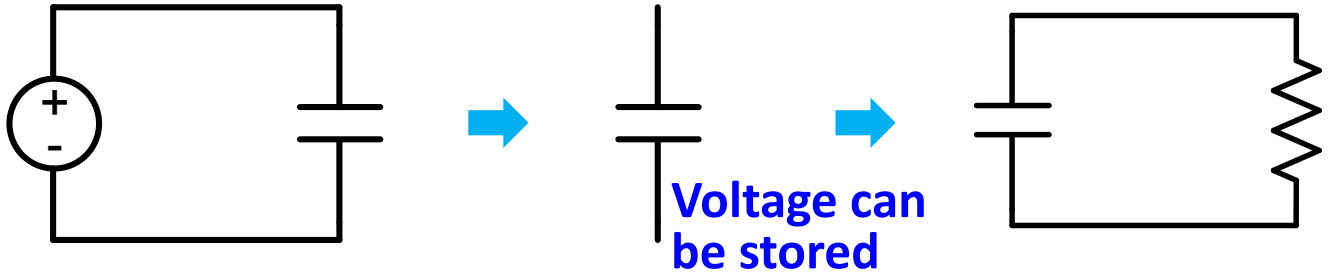
RLC

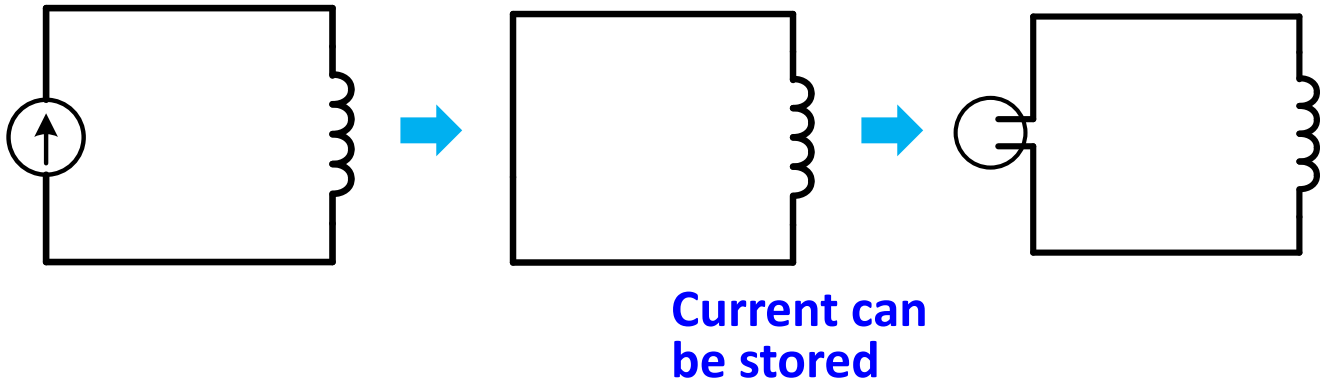
➔How can we form the differential equations in RLC circuit?

➔How can we solve the equation to determine the “response”?

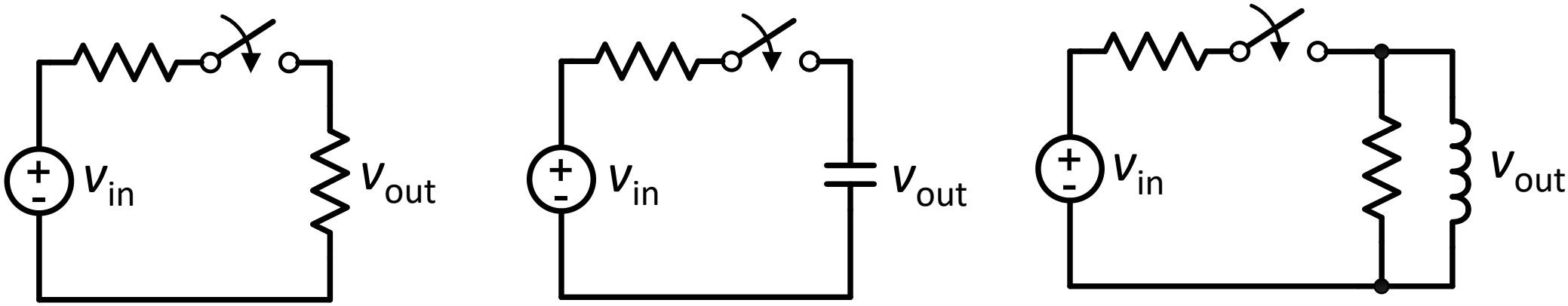
Difficulties for Handling C & L

- C and L can store energy in terms of electric field and magnetic field, respectively.





Initial condition Matters



➔**How does the response vary according to the initial condition?**

Chap. 9

The Complete Response of Circuits with Two Energy Storage Elements (1)

Hanwool Jeong

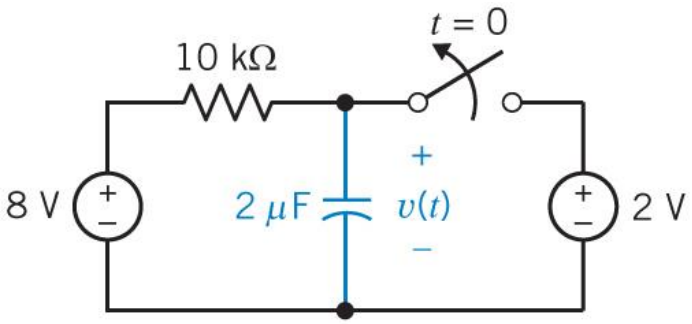
hwjeong@kw.ac.kr

Differential Equation for Circuits with Two Energy Storage Elements

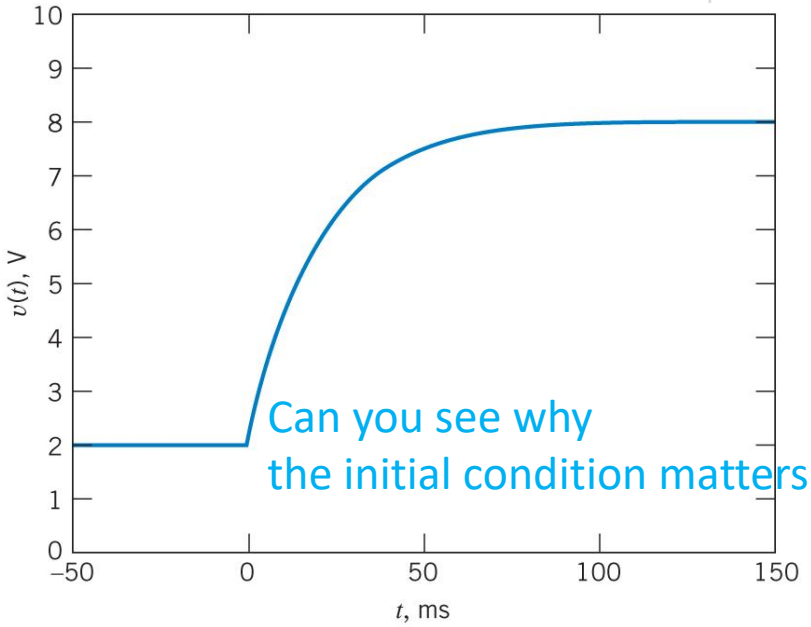
- ✓ Obtaining differential equation of a circuit

First-Order Circuit With a Capacitor

- Find the differential equation for v(t) in the following circuit



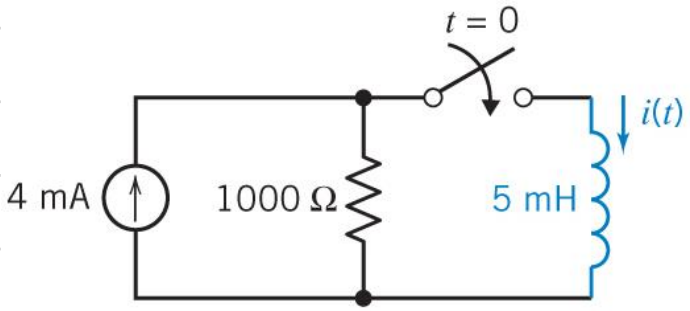
Tip: set v across a cap as the variable



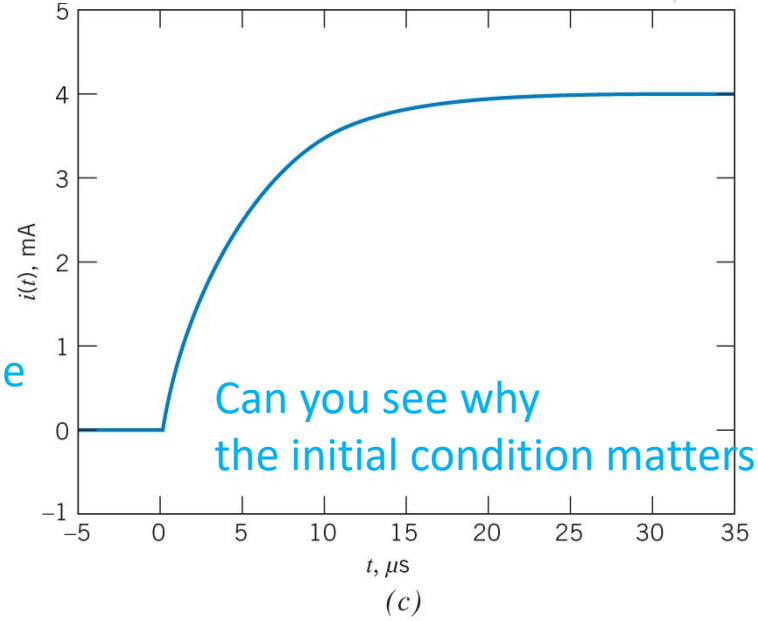
Can you see why
the initial condition matters?

First-Order Circuit With an Inductor

- Find the differential equation for i(t) in the following circuit



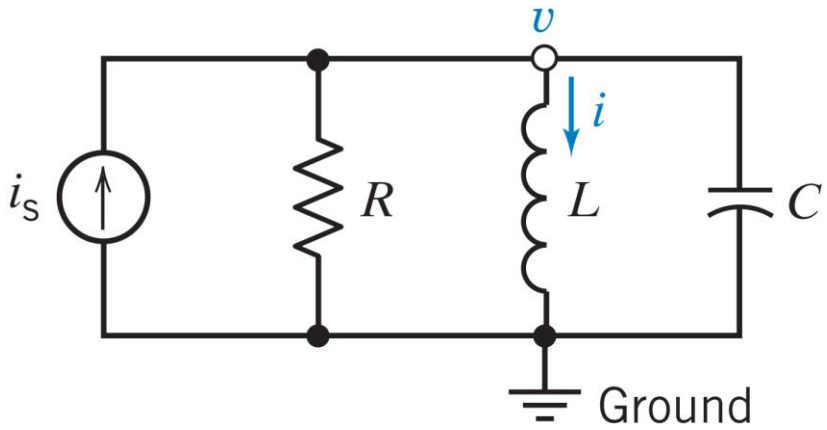
Tip: set *i* thru an inductor as the variable



Can you see why
the initial condition matters?

Parallel RLC Circuit

- Input : indep. Current source, i_s
- Output : current @ inductor L, i



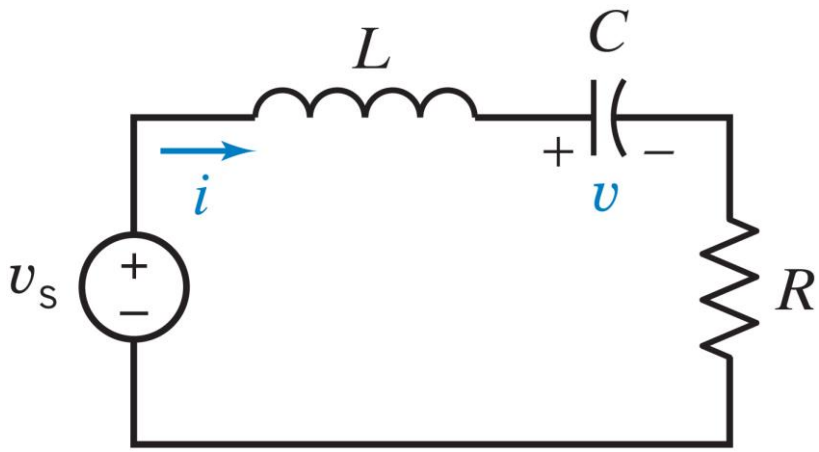
$$\frac{v}{R} + i + C\frac{dv}{dt} = i_s$$

$$v = L\frac{di}{dt}$$

$$\frac{L}{R}\frac{di}{dt} + i + CL\frac{d^2i}{dt^2} = i_s$$

Series RLC Circuit

- Input : indep. Voltage source, v_s
- Output : voltage @ capacitor, v



$$L\frac{di}{dt} + v + Ri = v_s$$

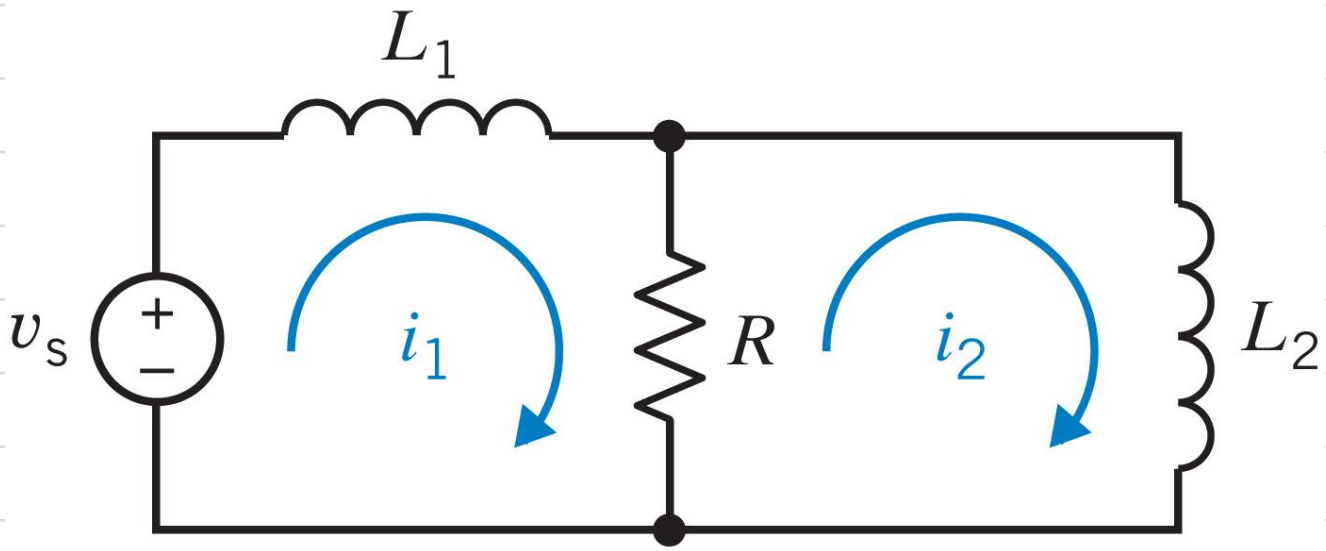
$$C\frac{dv}{dt} = i$$

$$C\frac{d^2v}{dt^2} + \frac{v}{L} + \frac{RC}{L}\frac{dv}{dt} = \frac{v_s}{L}$$

Direct Method for Obtaining 2nd Diff Eq.

Step 1	Identify the first and second variables, x_1 and x_2. These variables are capacitor voltages and/or inductor currents.
Step 2	Write one first-order differential equation, obtaining $\frac{dx_1}{dt} = f(x_1, x_2)$.
Step 3	Obtain an additional first-order differential equation in terms of the second variable so that $\frac{dx_2}{dt} = Kx_1$ or $x_1 = \frac{1}{K}\frac{dx_2}{dt}$.
Step 4	Substitute the equation of step 3 into the equation of step 2, thus obtaining a second-order differential equation in terms of x_2.

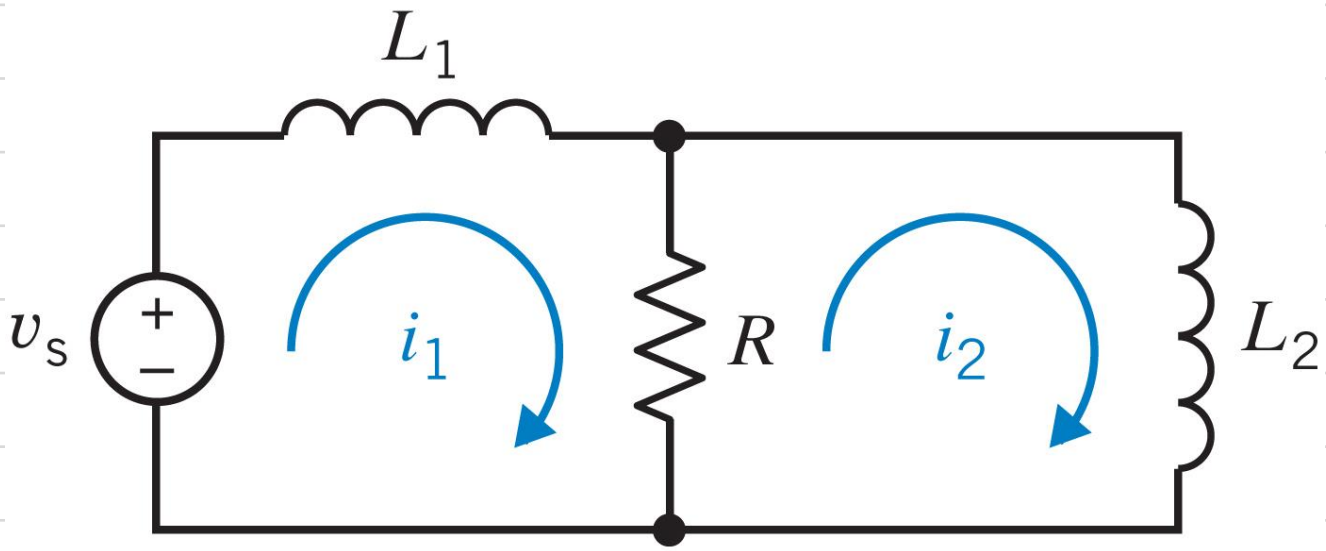
Circuits With Two Inductors



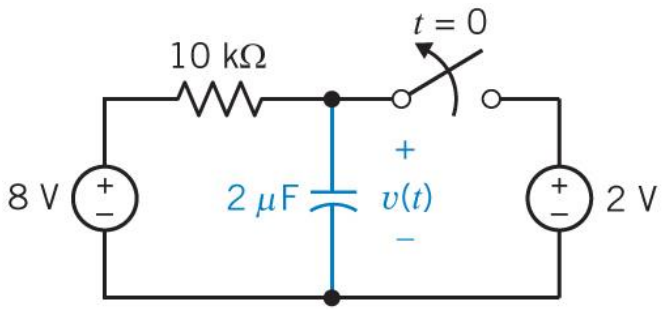
Operator-based Method

Step 1	Identify the variable x_1 for which the solution is desired.
Step 2	Write one differential equation in terms of the desired variable x_1 and a second variable, x_2.
Step 3	Obtain an additional equation in terms of the second variable and the first variable.
Step 4	Use the operator $s = d/dt$ and $1/s = \int dt$ to obtain two algebraic equations in terms of s and the two variables x_1 and x_2.
Step 5	Using Cramer's rule, solve for the desired variable so that $x_1 = f(s, \text{sources}) = P(s)/Q(s)$, where $P(s)$ and $Q(s)$ are polynomials in s.
Step 6	Rearrange the equation of step 5 so that $Q(s)x_1 = P(s)$.
Step 7	Convert the operators back to derivatives for the equation of step 6 to obtain the second-order differential equation.

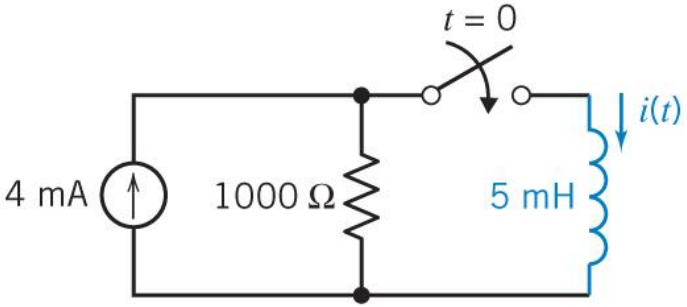
Revisit; Circuits With Two Inductors



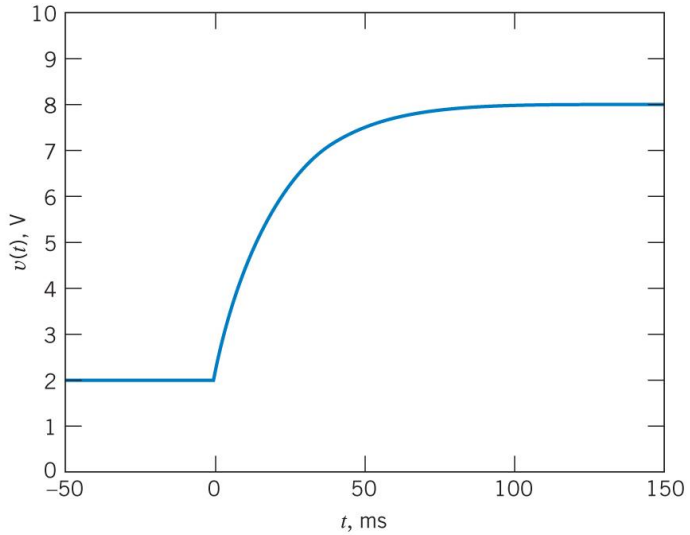
Need to Include Initial Conditions

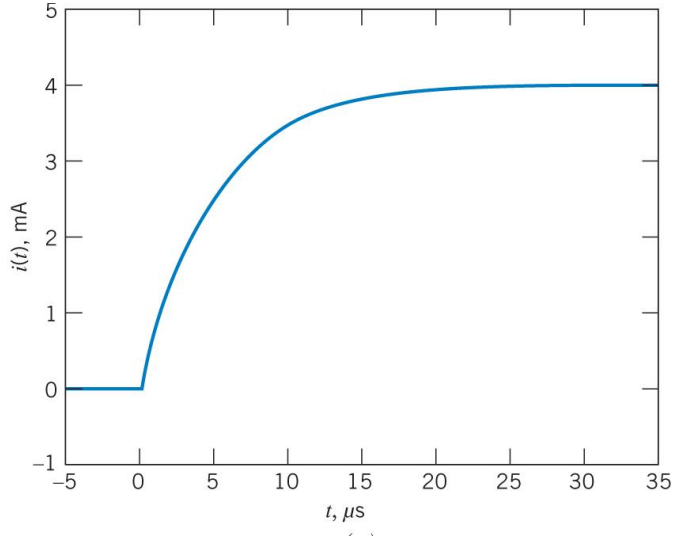


v + RC(dv/dt) = 8



i + (L/R)(di/dt) = 4

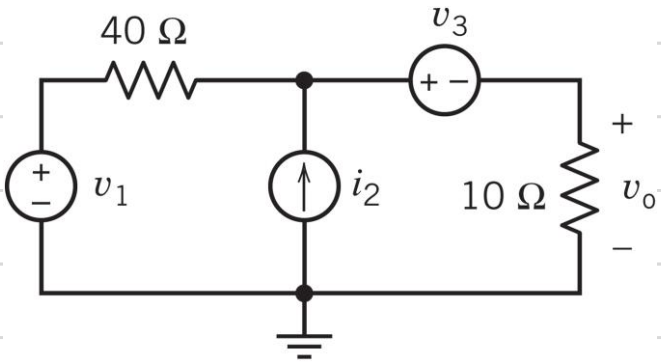




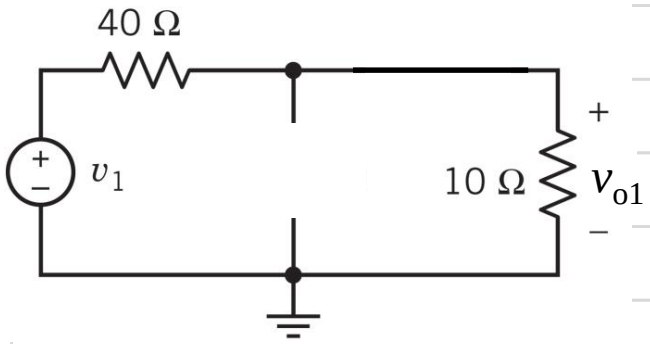
➔ **How can we derive the solution of differential equation w/ considering the initial condition?**

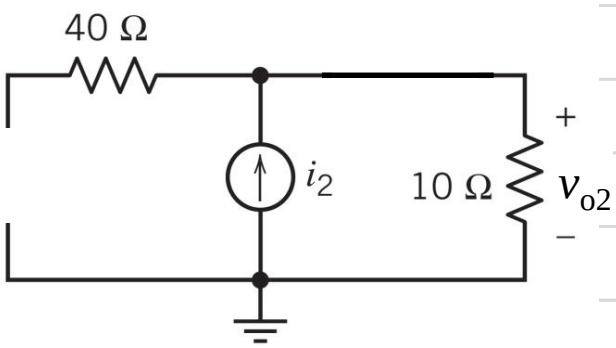
Revisit Superposition

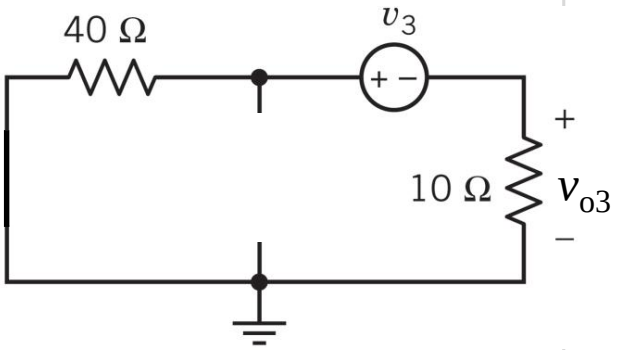
- Express the output current v_O in terms of three inputs, v_1, i_2 and v_3.



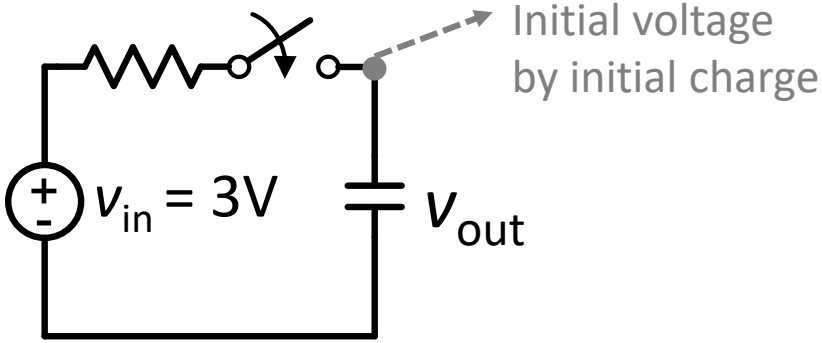
- Using superposition,

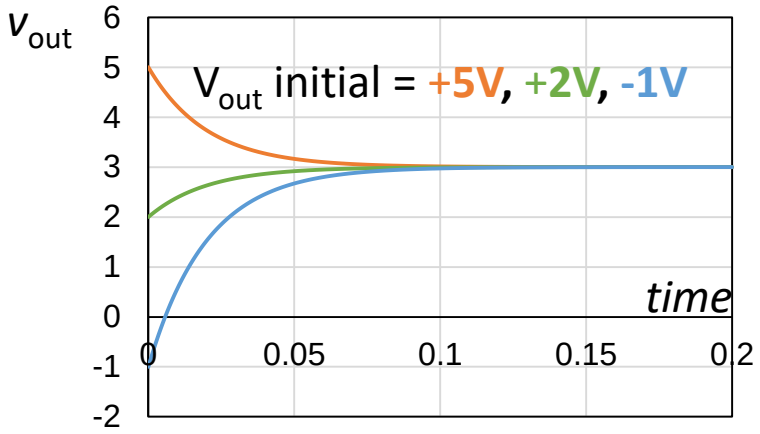






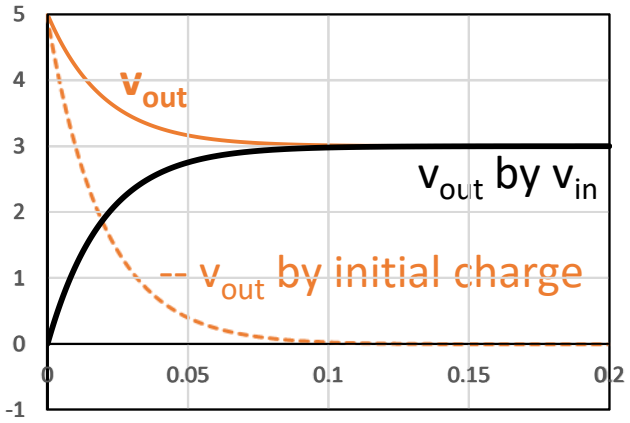
Handling Initial Condition?

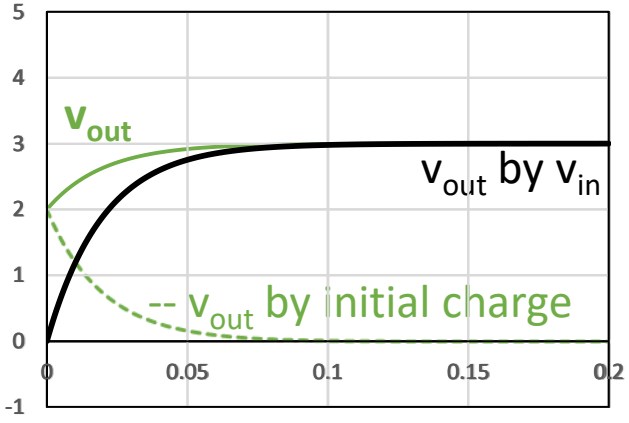


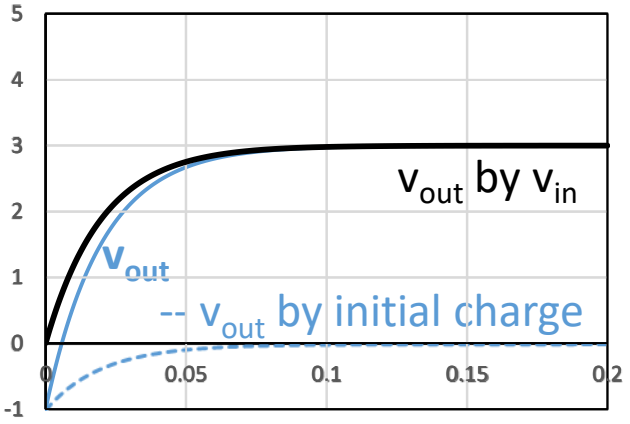


Initially stored energy is kind of energy source

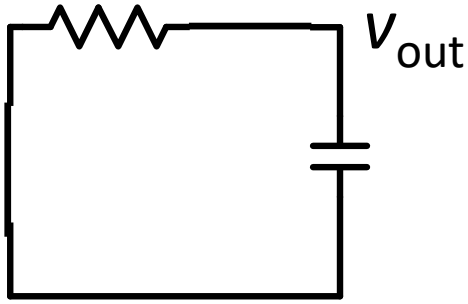
➔ **Why don't we utilize superposition as V/I source?**

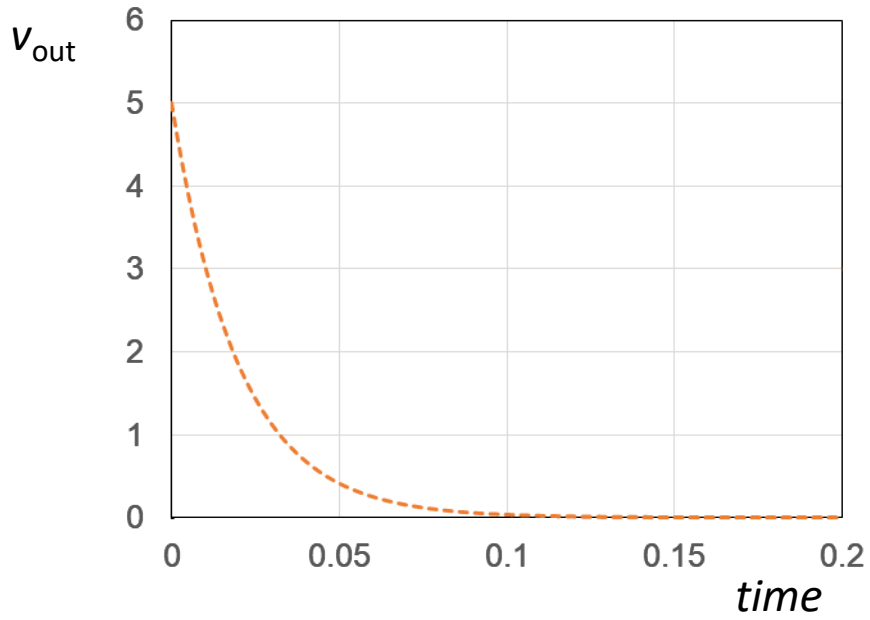






Response by Initial Condition?



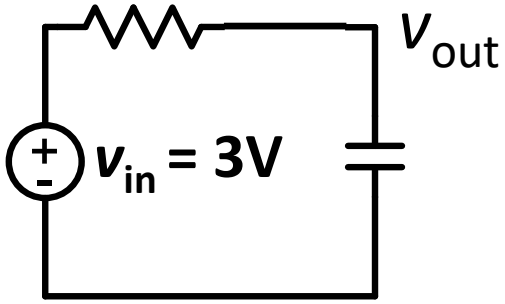


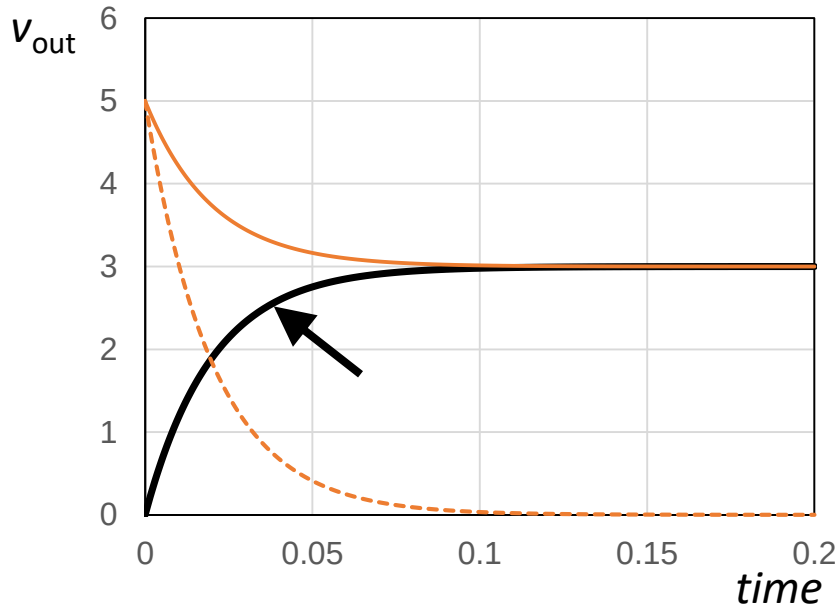
We just see how the circuit goes naturally.

➔ **Natural response**

: Can be expressed with the single format of equation

Response by Input Source





The circuit is forced to response to follow the source

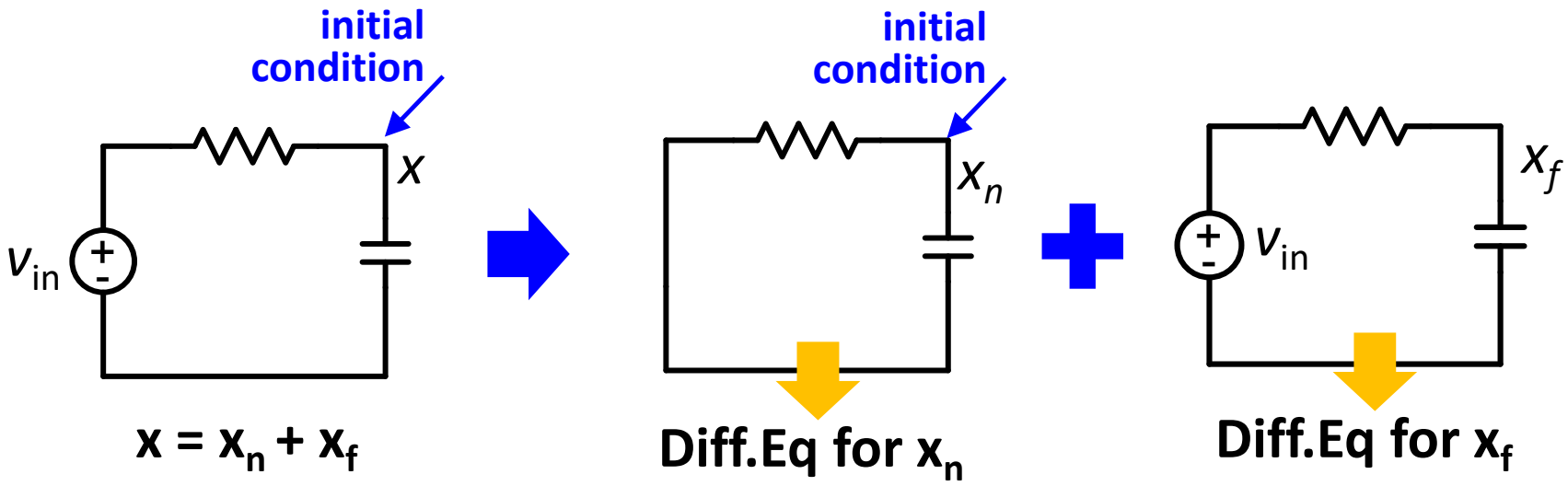
➔ **Forced response**

Complete Response

- Complete response = natural response + forced response
  - **Natural response** is the general solution of diff. equation when input is zero.
  - **Forced response** is a particular solution of the differential equation representing the circuit.

$$x = x_n + x_f$$

➔ **Now, we are ready to derive the circuit response!!**



Checkpoints

- How can we form the differential equations in RLC circuit?
- How can we consider the initial conditions of C & L?
- Coming up next: How can we solve the equation to determine the “response”?

Solution of the Second-Order Differential Equation – The Natural Response

- ✓ Characteristic equation & Natural frequency

Format of 2nd-Order Diff Eq

- Two irreducible energy storage elements can be represented by a second-order differential equation of the form,

Circuit response

Circuit Input

$$a_2 \frac{d^2 x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_0 x(t) = f(t)$$

- The complete response x(t) is given by

$$x = x_n + x_f$$

The term "(t)" is omitted

where x_n is natural response and x_f is forced response

➔ **First, we will focus on x_n**

By setting f(t) = 0 in RHS of diff. Eq.

Natural Response & Characteristic Eq.

- Natural response x_n will satisfy

$$a_2 \frac{d^2 x_n}{dt^2} + a_1 \frac{dx_n}{dt} + a_0 x_n = 0$$

$$\Rightarrow \quad x_n = Ae^{st}$$

- A and s are to be determined.

$$a_2 A s^2 e^{st} + a_1 A s e^{st} + a_0 A e^{st} = 0$$

$$\boxed{\left(a_2 s^2 + a_1 s + a_0\right)} = 0$$

Characteristic Equation

- Roots of the characteristic equation contain all the information necessary for determining the character of the natural response.

Solution of Characteristic Equation

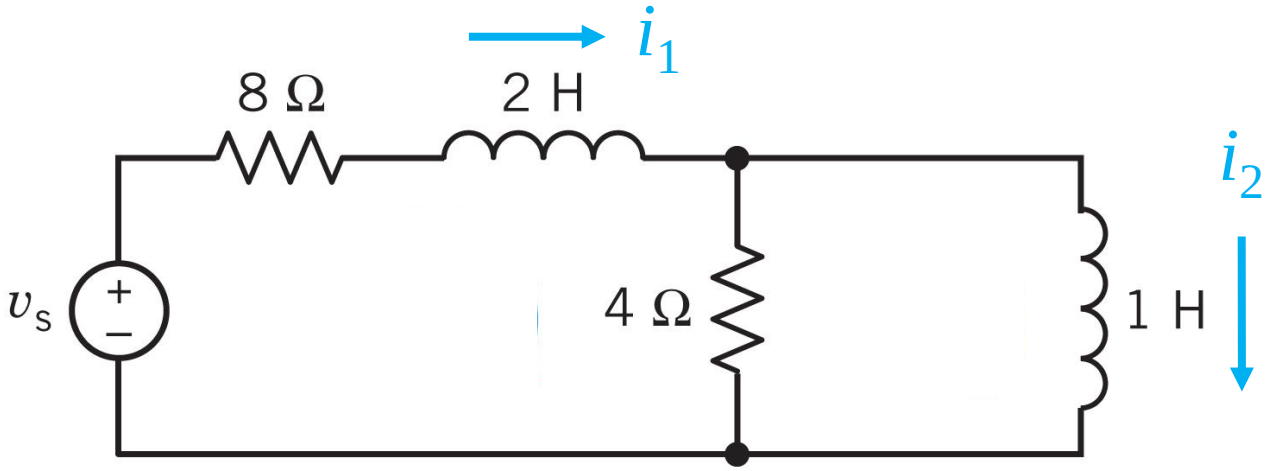
- Solutions of the characteristic equation are

$$s_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0}}{2a_2} \qquad s_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

- When there are two distinct roots, the natural response is of the form

$$x_n = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Example of Natural Response



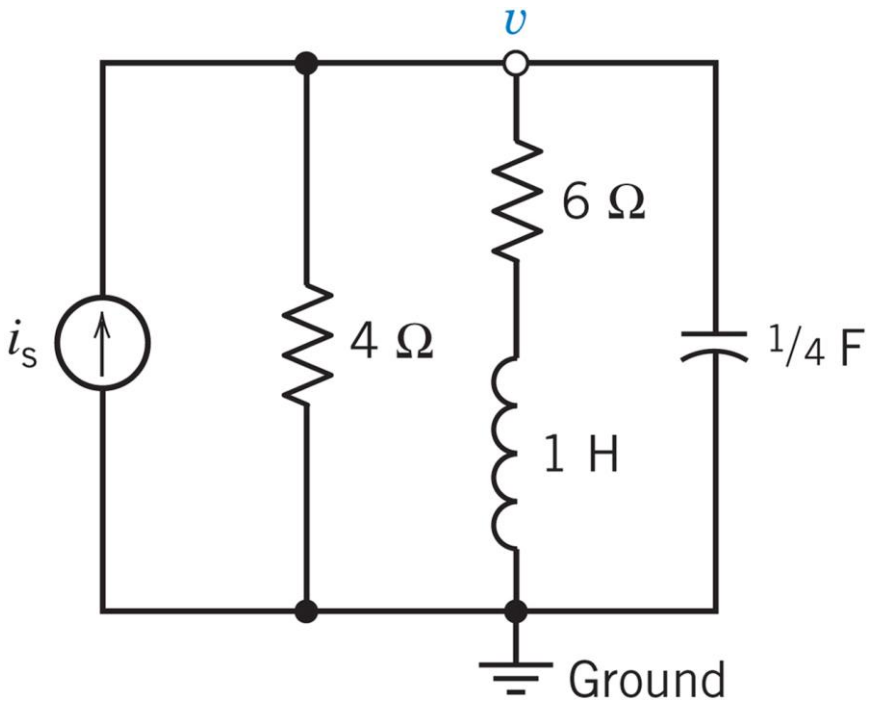
$$\text{KCL}: \mathrm{i}_1 = \mathrm{i}_2 + \frac{di_2}{dt}/4 \qquad \text{KVL}: \quad 8i_1 + 2\frac{di_1}{dt} + \frac{di_2}{dt} = v_S$$

$$8\left(i_2 + \frac{di_2}{dt}/4\right) + 2\frac{d\left(i_2 + \frac{di_2}{dt}/4\right)}{dt} + \frac{di_2}{dt} = v_S$$

$$\frac{d^2 i_2}{dt^2} + \left(16i_2 + 4\frac{di_2}{dt}\right) + 4\frac{di_2}{dt} + 2\frac{di_2}{dt} = v_S$$

➔ **Why general term?**

Example of Natural Response (2)



Checkpoints

- We saw natural response for simple RLC circuit
- Characteristic equation?
- General term? Why?