Chap. 9

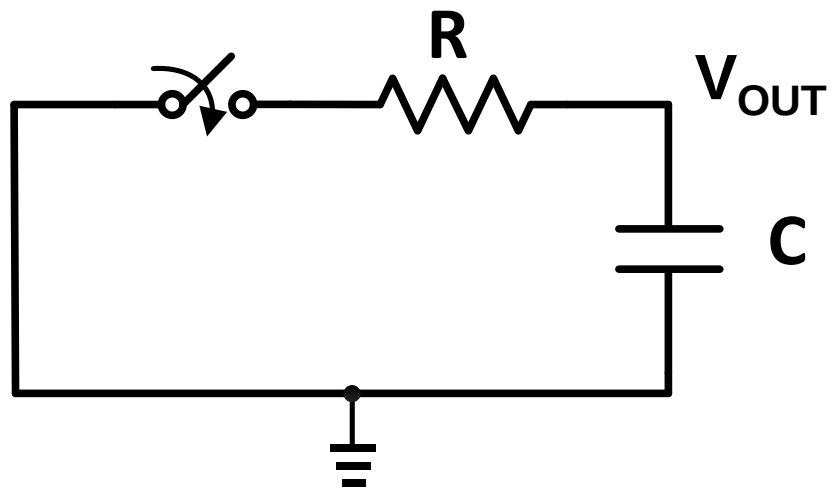
The Complete Response of Circuits with Two Energy Storage Elements (3)

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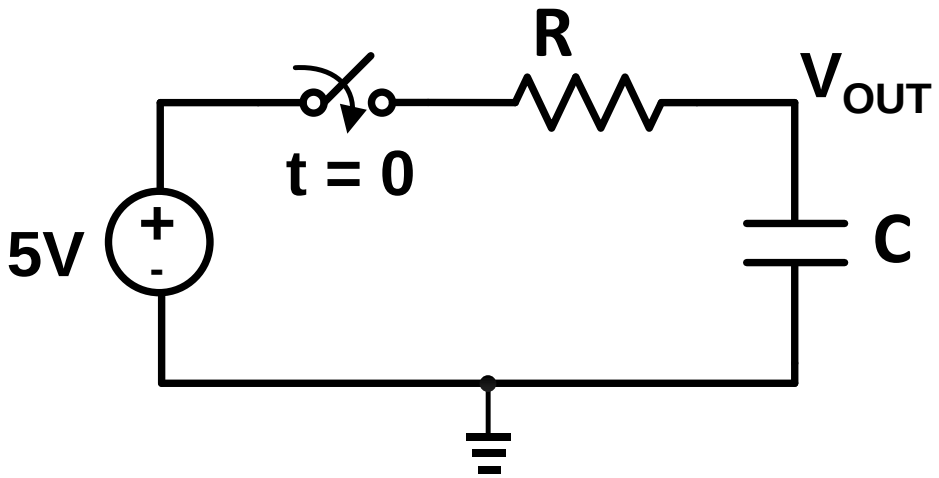
Revisit Simple RC Circuit

- As a energy storage element, V_{OUT} can have store voltage.
- Then, as switch closes, V_{OUT} decreases gradually.

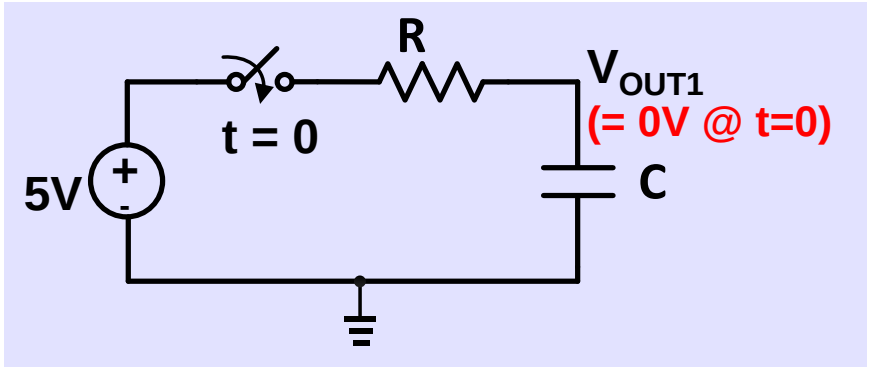


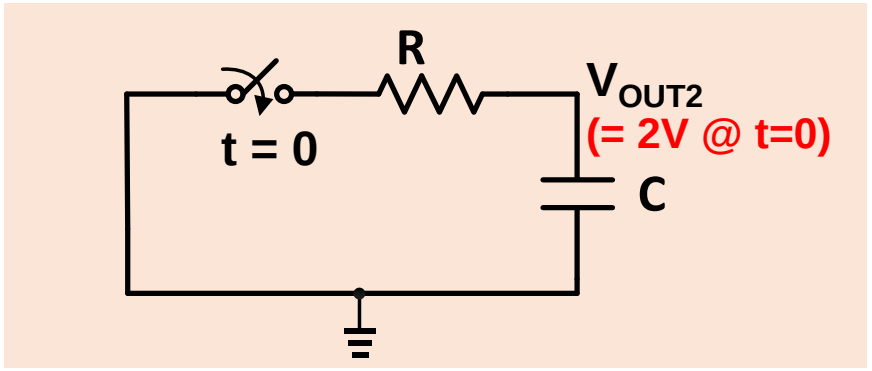
Revisit Simple RC Circuit w/ Forced Input

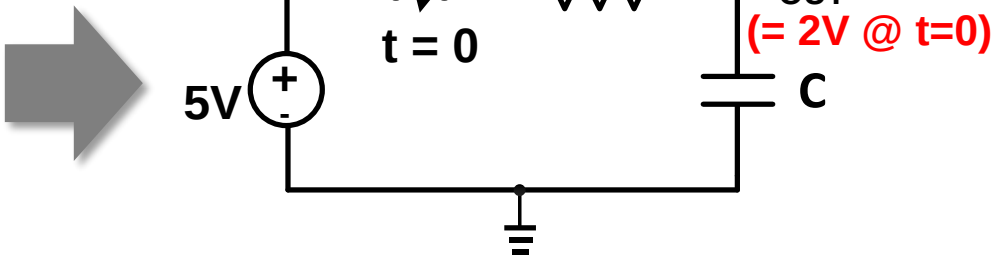
- We can say that $\mathbf{V_{OUT}}$ would be ended up as 5V.
- However, how about the beginning of $\mathbf{V_{OUT}}$?
- At the beginning, V_{OUT} can have **any value**.
  ➔ Uniqueness of energy storage element. **Like V_{OUT} = 2V or 7V @ t =0**



Superposition;
Simple + Simple = Complex

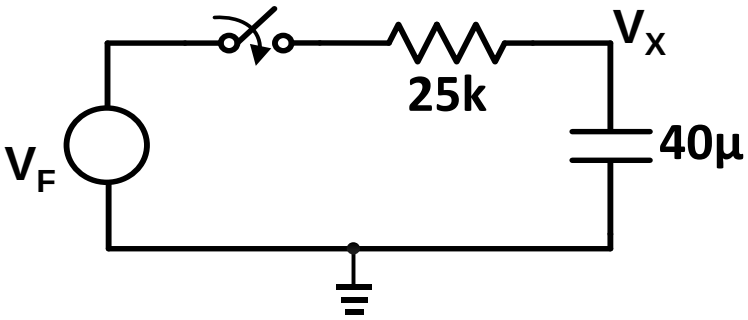


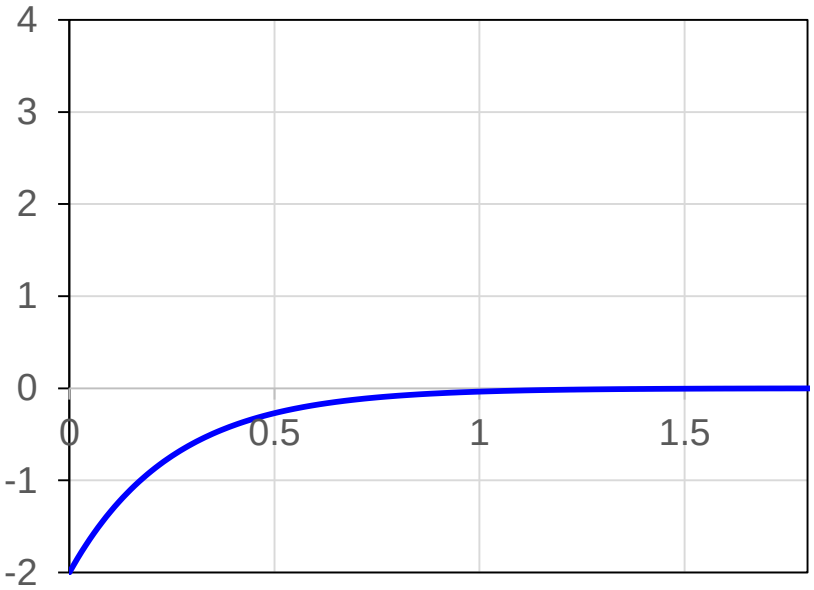


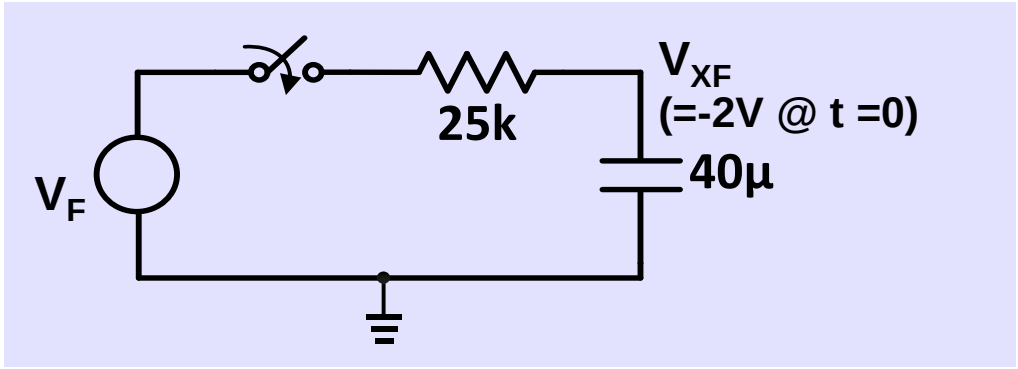


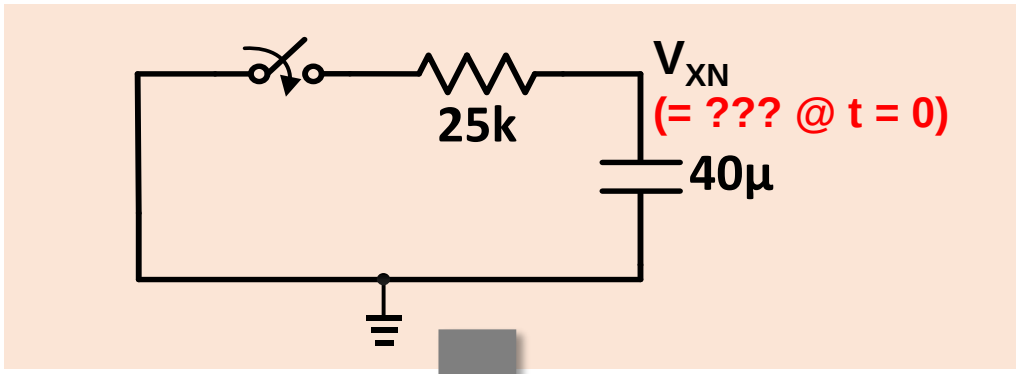
RC Example

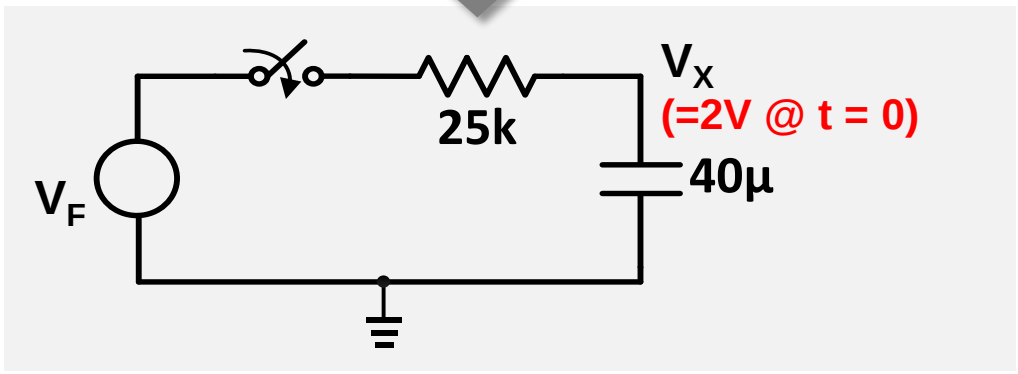
- When $V_F = 6e^{-4t}$, how would V_X change?
- If V_X is initially -2V, the situation is simple. But, what if V_X have different initial condition? Like **V_X = 2V @ t = 0.**

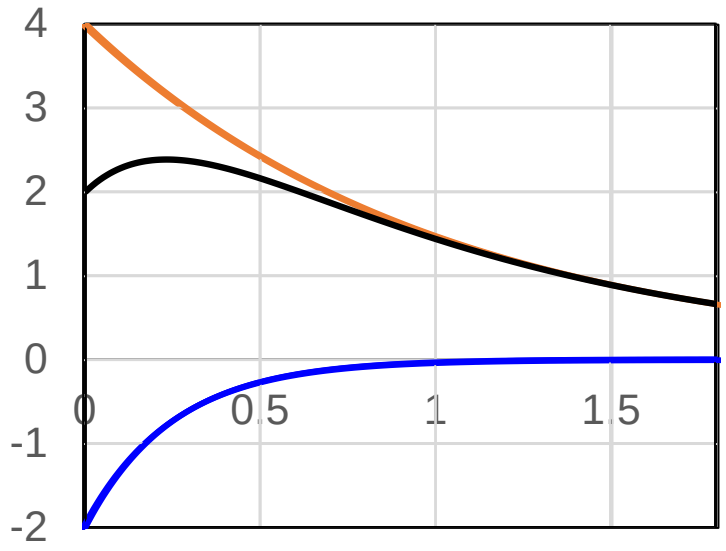





V_{XF}
(=-2V @ t =0)
25k
40µ
V_F

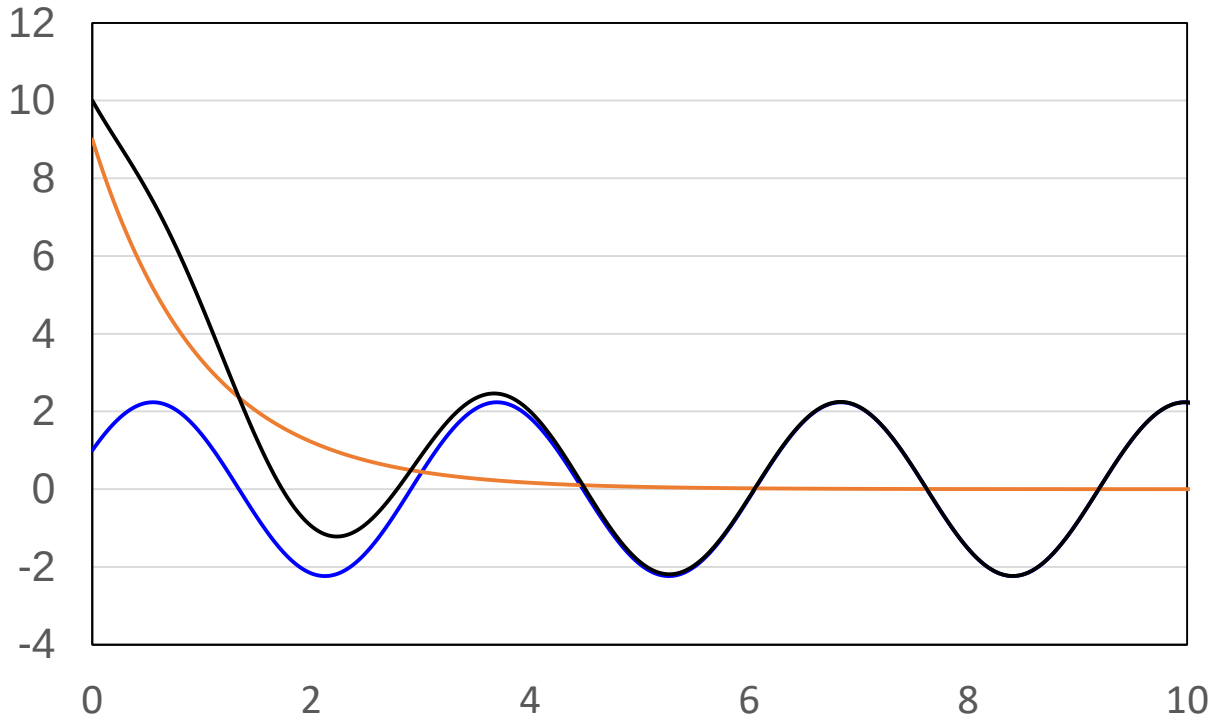

V_{XN}
(= ??? @ t = 0)
25k
40µ


V_X
(=2V @ t = 0)
25k
40µ
V_F


4
3
2
1
0
-1
-2
0
0.5
1
1.5

Sinusoidal Example

- When $V_F = 5\cos 2t$



Complete Response of RLC Circuits

- ✓ 9.7 Forced Response of an RLC Circuit
- ✓ 9.8 Complete Response of an RLC Circuit

Natural vs. Forced

- Differential equation for 2nd order circuit is

$$\frac{d^2x}{dt^2} + a_1\frac{dx}{dt} + a_0x = f(t)$$

- The forced response x_f should satisfy the following equation

$$\frac{d^2x_f}{dt^2} + a_1\frac{dx_f}{dt} + a_0x_f = f(t)$$

- How? ➔ Case Study
  - If the forcing function f(t) is a constant, we expect the forced response also to be a constant (why?)
  - How about $Be^{-\alpha t}$? ➔ $x_f = De^{-\alpha t}$ (why?)
  - How about sinusoidal function f(t)? ➔ $x_f = M\sin\omega t + N\cos\omega t$

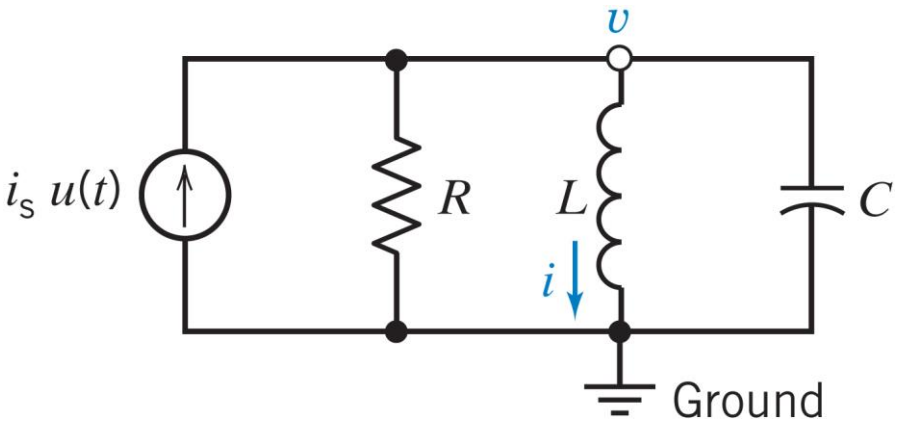
Assumed Solution (Response)

$$\frac{d^2x_f}{dt^2} + a_1\frac{dx_f}{dt} + a_0x_f = f(t)$$

FORCING FUNCTION	ASSUMED RESPONSE
K	A
Kt	$At + B$
Kt^2	$At^2 + Bt + C$
$K \sin \omega t$	$A \sin \omega t + B \cos \omega t$
Ke^{-at}	Ae^{-at}

Example 9.7-1

- L=7H, R=7Ω, C=1/42F
- u(t)= step function, $i_s = 8e^{-2t}$
- Find the forced response



$$i + \frac{v}{R} + C\frac{dv}{dt} = i_s$$

$$v = L\frac{di}{dt}$$

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{i_s}{LC}$$

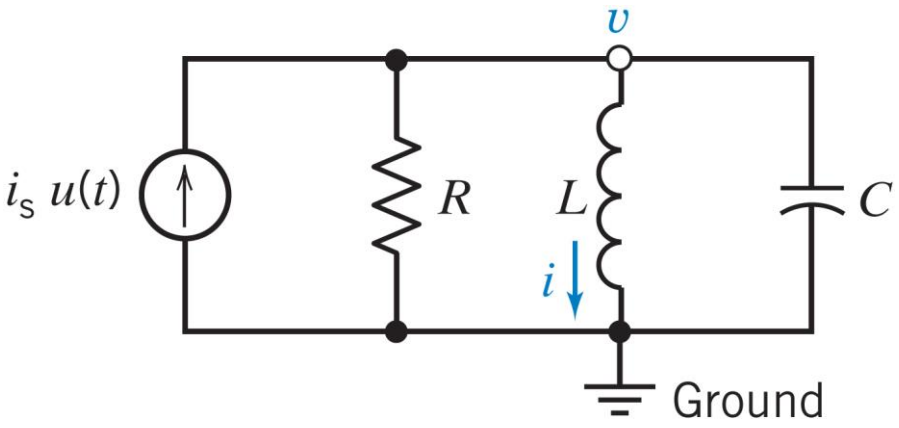
$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 6\,i = 48e^{-2t}$$

$$i_f = Be^{-2t}$$

Example 9.7-2

- L=7H, R=7Ω, C=1/42F
- u(t)= step function, $i_s = I_0$ (constant)
- Find the forced response

$$\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 6\,i = 6\,i_s$$



When the Forced Response has the Same Term with Natural Response

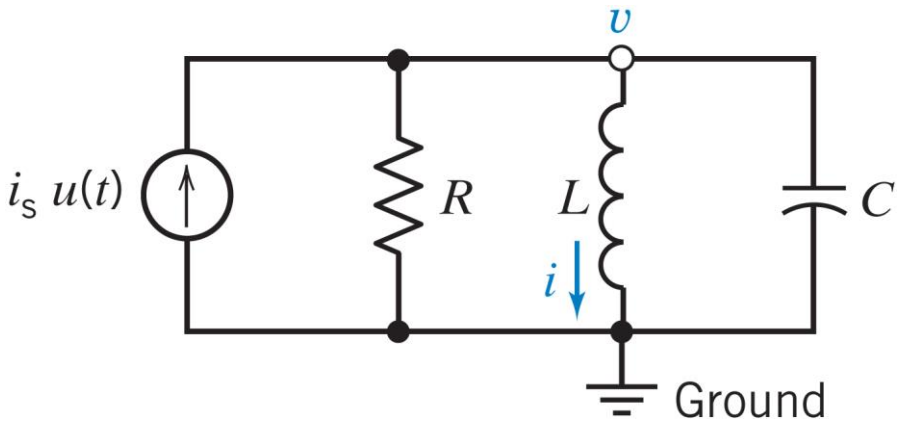
- $i_s = 3e^{-6t}$
- Find the forced response

$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 6\,i = 6\,i_s$$

$$\frac{d^2i}{dt^2} + 7\frac{di}{di} + 6i = 18\,e^{-6t}$$

- Natural response?

- Can $i_f = Be^{-6t}$ work?



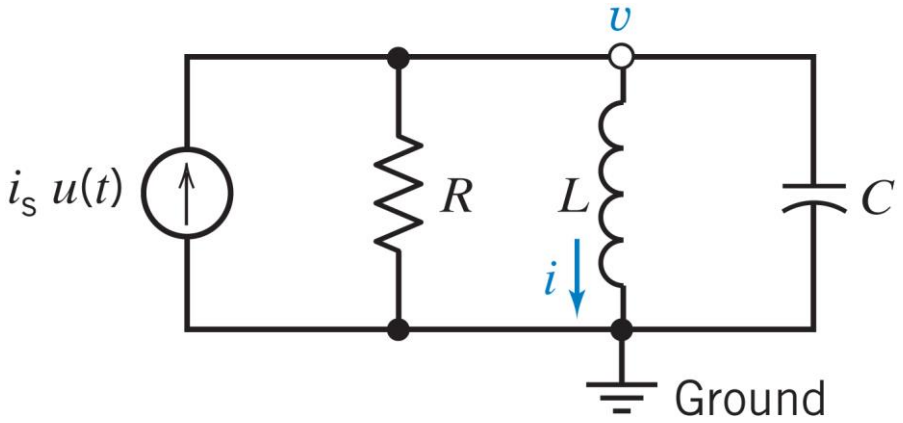
➔ We need another form of a forced response

When the Forced Response has the Same Term with Natural Response

- $i_s = 3e^{-6t}$

$$\frac{d^2i}{dt^2} + 7\frac{di}{di} + 6i = 18\,e^{-6t}$$

- Let's try $i_f = Bte^{-6t}$



$$B\left(-6e^{-6t} - 6e^{-6t} + 36t\,e^{-6t}\right) + 7B\left(e^{-6t} - 6t\,e^{-6t}\right) + 6Bt\,e^{-6t} = 18\,e^{-6t}$$

- B = $-\frac{18}{5}$, and $i_f = -\frac{18}{5}te^{-6t}$

- **In general, $x_f = t^p x_{n1}$**

Exercise 9.7-1

- Find the forced response when i_s = 6 + 2t

$$\frac{d^2 i}{dt^2} + 9\frac{di}{di} + 20i = 6i_s$$

Complete Response

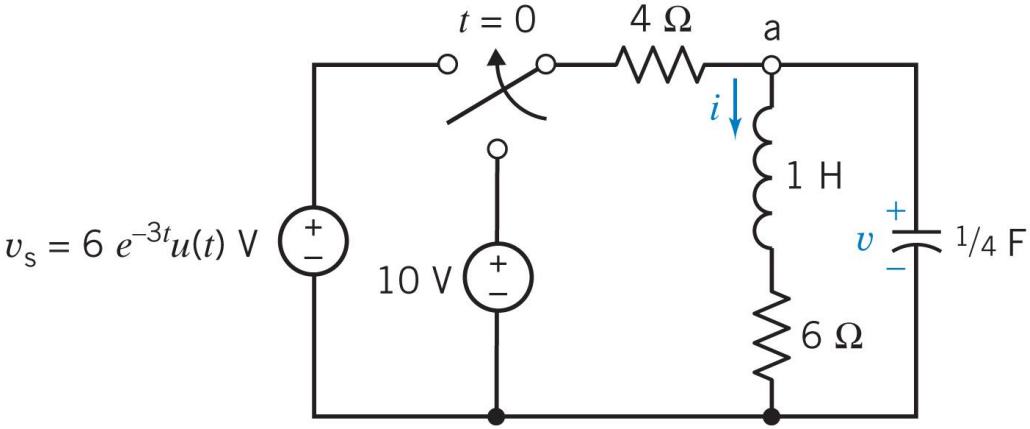
- Complete response : $x = x_n + x_f$
- Consider a circuit described by the following 2nd order diff. equation

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 6\,v_s \qquad \text{with } v_s = \frac{2}{3}e^{-t}\,V,\ v(0)= 10,\ dv(0)/dt = -2V/s$$

- Natural response
- Forced Response
- Complete response?

Example 9.8-1

- Find the complete response
- Circuit is at steady state at $t=0^-$



Example 9.8-1

