Chap. 10

Sinusoidal Steady-State Analysis (1)

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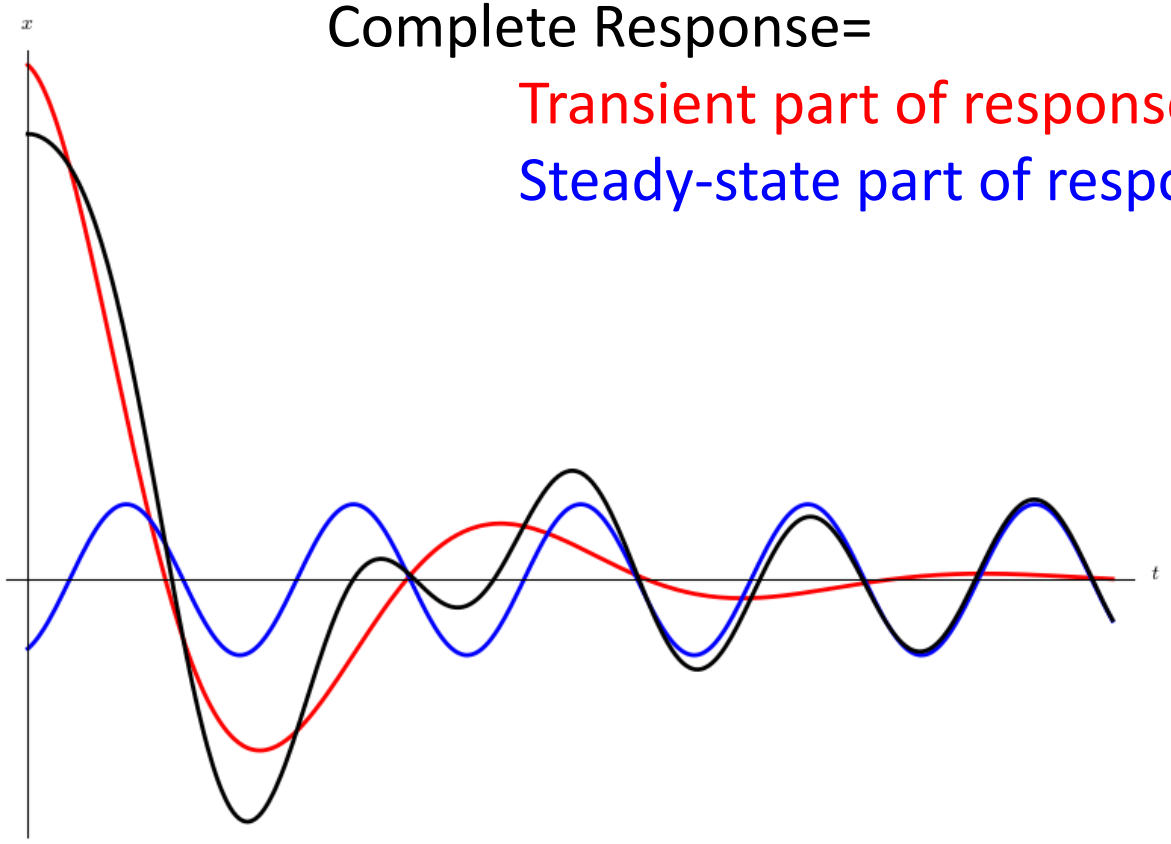
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Phasor for AC Circuit

- ✓ 10.2 Sinusoidal Sources
- ✓ 10.3 Phasors and Sinusoids

Transient & Steady State Response

- The output of circuit consists of 1) transient part & 2) steady-state part.
- Steady state response?

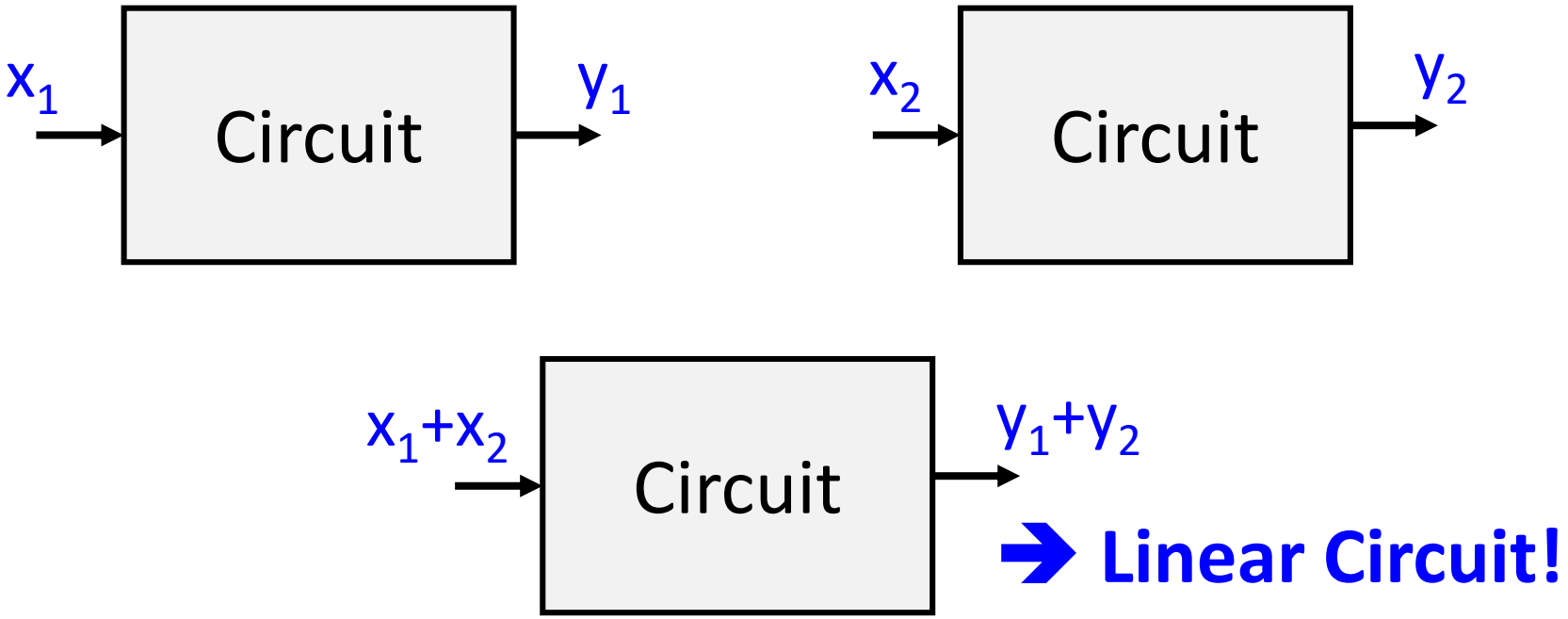


Complete Response=

Transient part of response +

Steady-state part of response

Linear Circuit



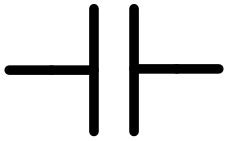
Is RLC Circuit Linear?



$v = i \times R$



$v = L \times di/dt$



$i = C \times dv/dt$

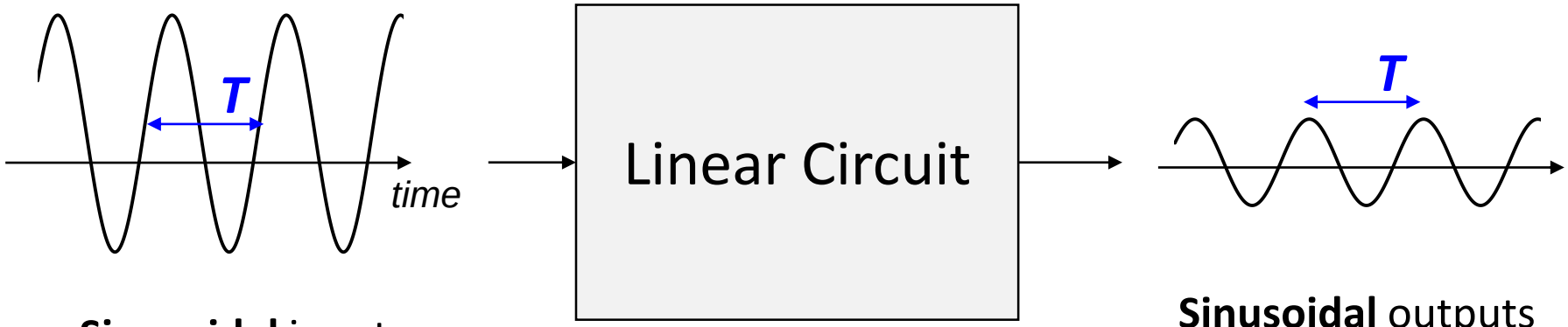
$v_1 \rightarrow C \times dv_1/dt = i_1$

$v_2 \rightarrow C \times dv_2/dt = i_2$

$v_1 + v_2 \rightarrow C \times d(v1+v_2)/dt = i_3$

Definition of AC Circuit

- Linear circuit with sinusoidal inputs that are at steady state are called **ac circuits**. ➔ First, we will cover the case of input having the same freq.
- Outputs of the circuit are also sinusoidal and have the same frequency with the input, at the steady state.



Sinusoidal inputs
@ steady state

$i(t) = 30\cos(\omega t)$ mA

$v(t) = 7\cos(\omega t)$ V

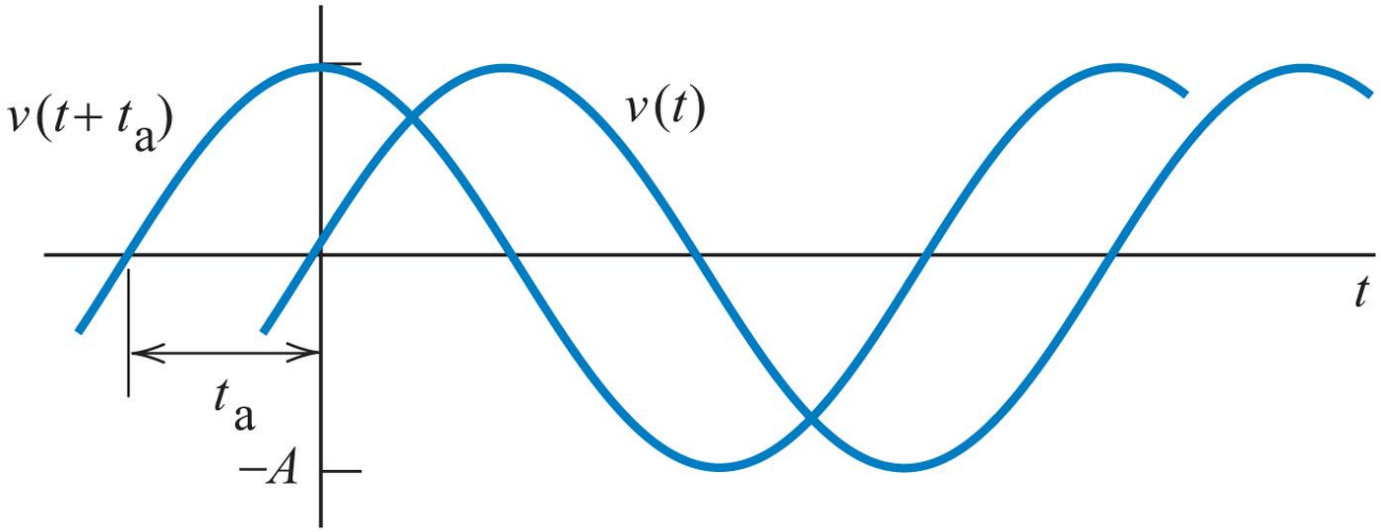
$$\omega = 2\pi f = \frac{2\pi}{T}$$

Sinusoidal outputs
@ steady state

$i_{OUT}(t) = 10\cos(\omega t + 30^o)$ mA

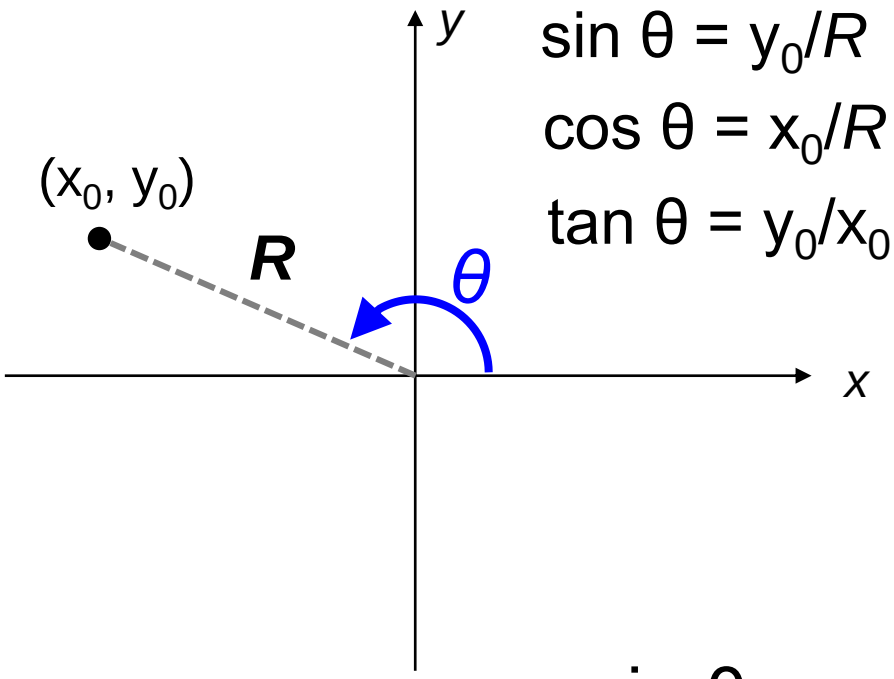
$v_{OUT}(t) = 3\cos(\omega t - 50^o)$ mA

Parallel Displacement in Graph



Sine vs. Cosine

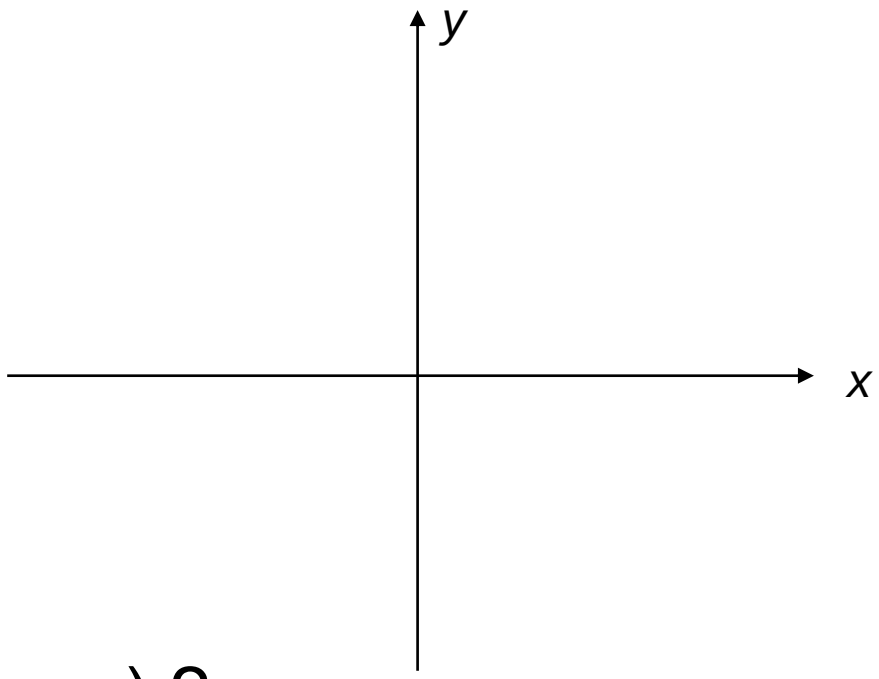
- Definition of sine, cosine and tangent function?



$\sin\theta = y_0/R$

$\cos\theta = x_0/R$

$\tan\theta = y_0/x_0$



$\sin\theta = \cos(\quad)$?

$\cos(-\theta) = \cos(\quad)$?

Features of Sine & Cosine Functions

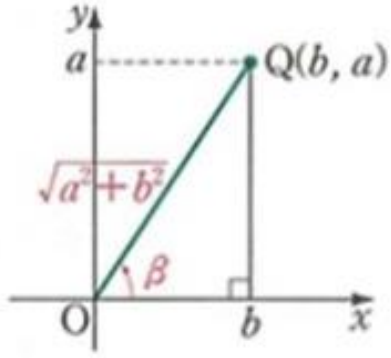
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

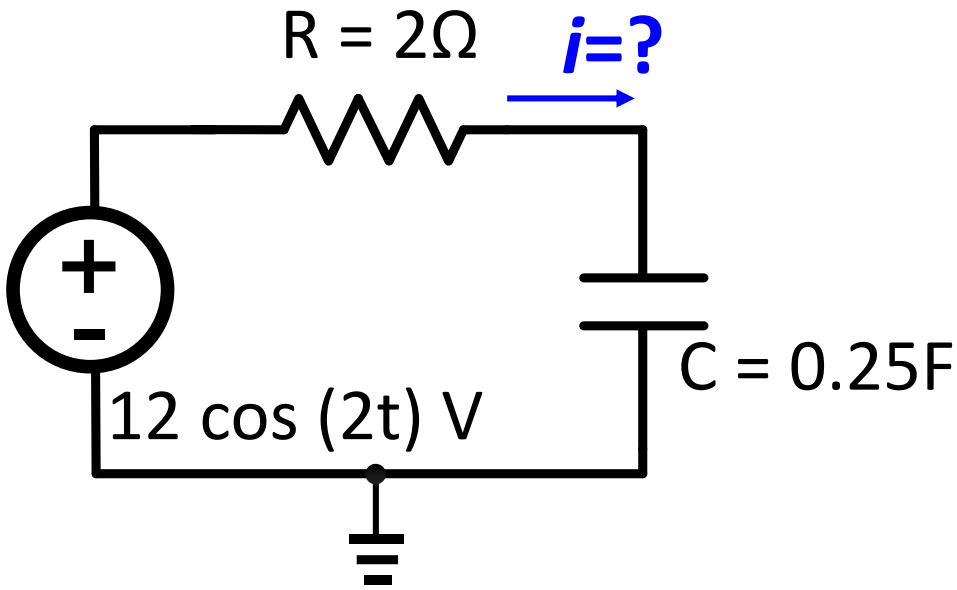
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

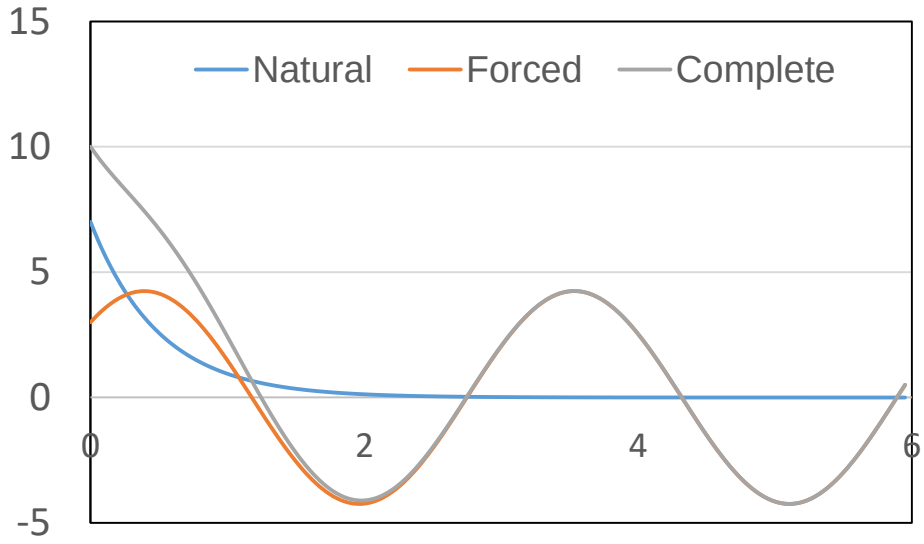
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\cos(\theta-\beta)$$



Simple RC Circuit Example





For stable RLC (AC) circuit, we can say

Forced response = steady state response

Natural response = transient response

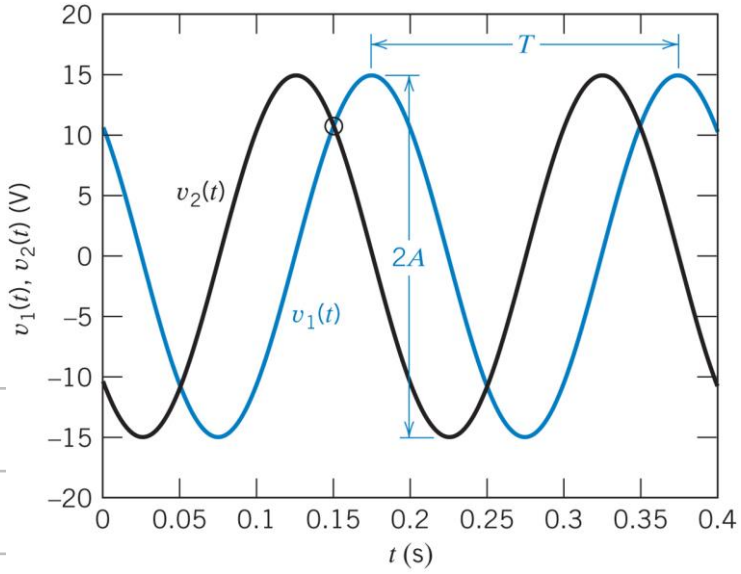
Example 10.2-1

- $v_1(t) = 10 \cos(200t+45^o)$, $v_2(t) = 8 \sin(200t - 75^o)$ when t is expressed in *ms*

➔ $v_2(t)$ is delayed with respect to $v_1(t)$ by ( )

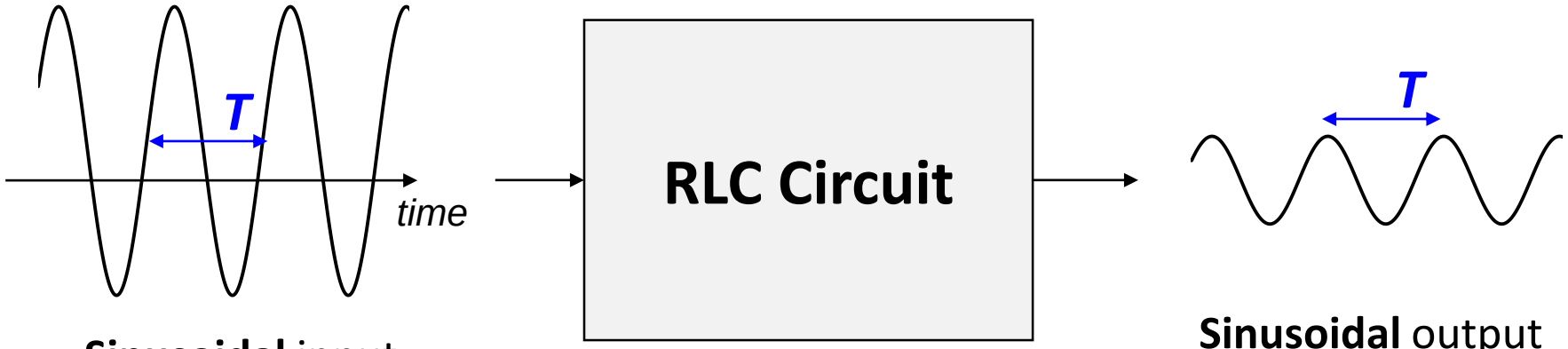
Example 10.2-2

- Determine the analytic representation



Revisit AC Circuit

- We will focus on RLC circuit!



Sinusoidal input
@ steady state

Sinusoidal output
@ steady state

$$v = i \times R$$

$$v = L \times di/dt$$

$$i = C \times dv/dt$$

All *v* & *i* are **sinusoidal** inputs w/ the **same frequency**

Sinusoidal w/ Same Freq?

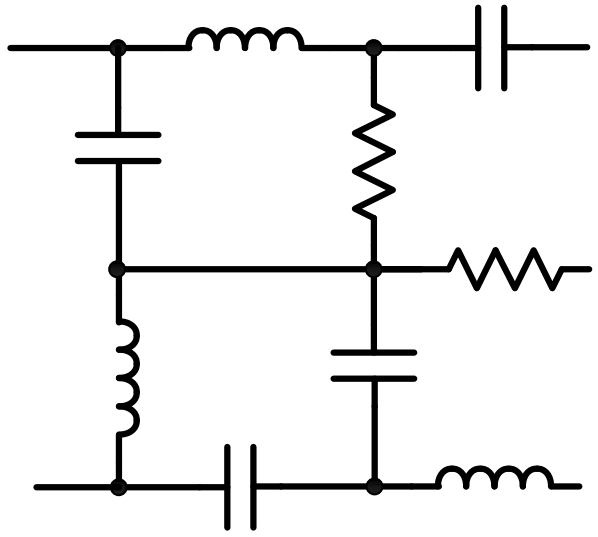
- We can only focus on magnitude and phase.

$$A\cos(\omega t+\theta)$$

Never changes

Only changing

➔ **(A, θ) only matters**



- ✓ All i & v are sinusoidal w/ same freq.
- ✓ They can be derived w/ differential & integrals

v = i×R **v = L× di/dt** **i = C× dv/dt**

In AC RLC Circuit,

- We can focus (A, θ) only
  - ➔ **So let's express v & I w/ (A, θ)**
  - ➔ What we want is to establish a new system (but more simple)
- All *i*&*v* can be derived with differential or integral
- The target system should follow

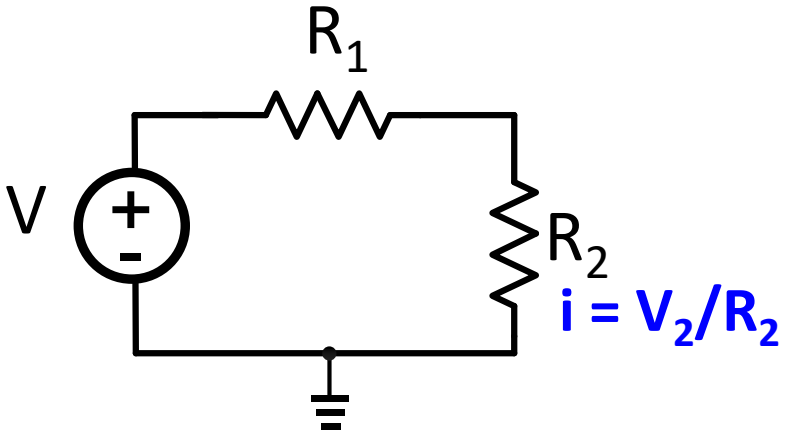
Differential of (A, θ) ➔ (ωA, θ+**90°**)

$A\cos(\omega t+\theta)$ ➔ $\omega A\cos(\omega t+\theta+90^{o})$

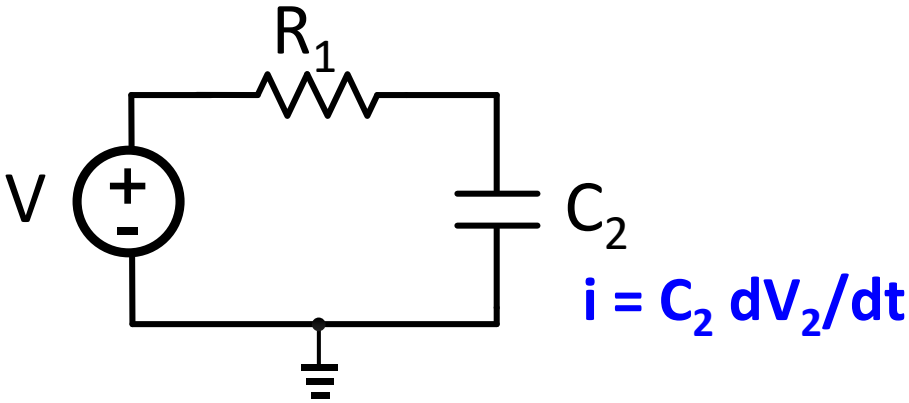
Integral of (A, θ) ➔ (A/ω, θ-**90°**)

Replacing Calculus w/ Arithmetic

- Which is more difficult? **Why?**



Calculus X



Calculus O

But what if AC circuit?
That is, Sinusoidal?

V_2
← ×ω
← +90°
← ×C_2

Can't we derive $i = V_2 / \square_{C2}$ as we do in $i = V_2/R_2$, with well defining $\square_{C2}$?

Our Target

$I = V/\square$ & $V = I \times \square$ for L and C as well

➔ So we can use KVL and KCL freely

- If we want to do "calculation" above, there is a single closed format for expressing **I** or **V**, where that format should contain (A, θ) information.

Original world

$i_C = C \times dv_C/dt$

Our world

$I_C = V_C/\square_C$

V_C
← $\times \omega$
← $+90^o$
← $\times C$

Multiplication & Division? ➔ Addition or Subtraction

➔ We can locate θ at the exponent!

That Leads to

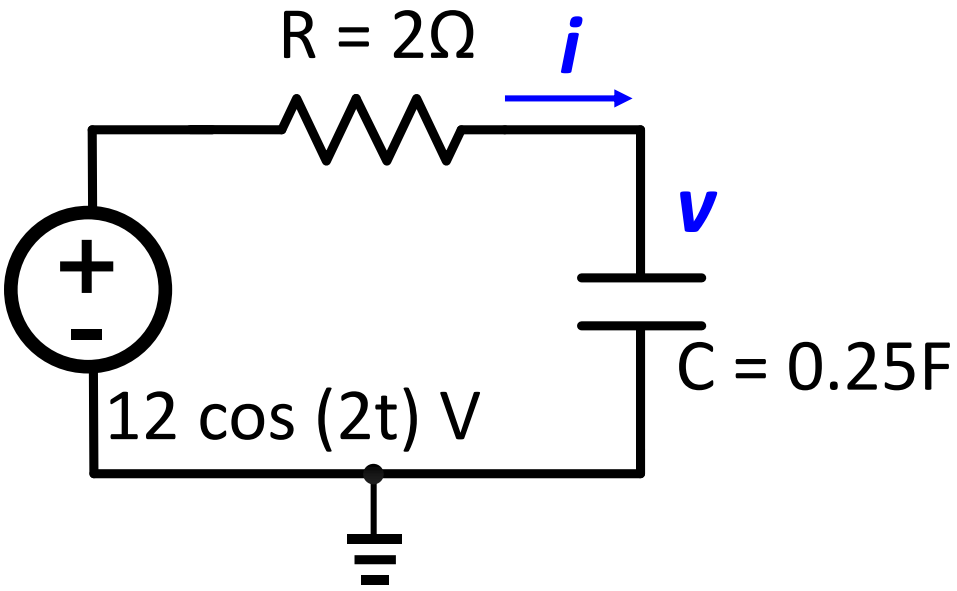
- Now we form a new system expresses V and I as:

Original world	**Our world**
$v(t) = A_{\mathrm{V}} \cos(\omega t + \theta_{\mathrm{V}})$	$V = A_{\mathrm{V}} x^{\theta_V}$
$i(t) = A_{\mathrm{I}} \cos(\omega t + \theta_{\mathrm{I}})$	$I = A_{\mathrm{I}} x^{\theta_I}$
R	R
C	$1/\omega C x^{90^{\circ}}$
L	$\omega L x^{90^{\circ}}$

- Should we use unknown variable *x*?

➔ We should guarantee the uniqueness, we can separate A and θ after the calculation.

Revisit RC Circuit Example



$v(t) = 6\cos 2t + 6\sin 2t$

$= 6\sqrt{2}\cos(2t-45^o)$

$i(t) = 3\cos 2t - 3\sin 2t$

$= 3\sqrt{2}\cos(2t+45^o)$

How can we merge (±) different phased expressions?

We can use constant for x! But..

Is there any method that guarantees uniqueness while we can use constant x?

Uniqueness?

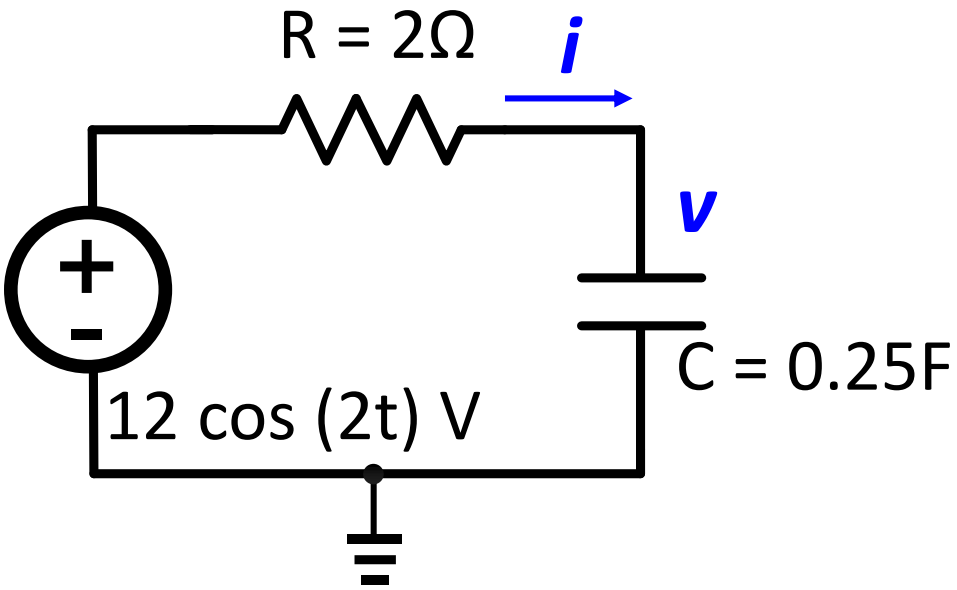
- $2 \times 3^3 = 6 \times 3^2$ ➔ Can you distinguish A & θ?
- How about this?

$$\mathbf{2 \times e^{j3}}$$

➔ **± Cannot remove Distinguishability b/w real and imaginary number**

➔ **It is why we use complex number in various fields!**

Revisit RC Circuit Example



$v(t) = 6\cos 2t + 6\sin 2t$

$= 6\sqrt{2}\cos(2t-45^o)$

$i(t) = 3\cos 2t - 3\sin 2t$

$= 3\sqrt{2}\cos(2t+45^o)$

Conclusion

Original world

$v(t) = A_V \cos(\omega t + \theta_V)$

$i(t) = A_I \cos(\omega t + \theta_I)$

R

C

L

Our world

$V = A_v \, exp(j\theta_V)$

$I = A_I exp(j\theta_I)$

R

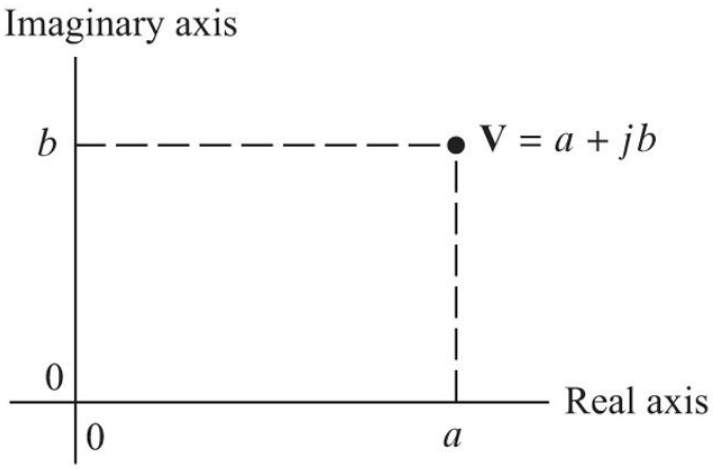
$1/j\omega C$

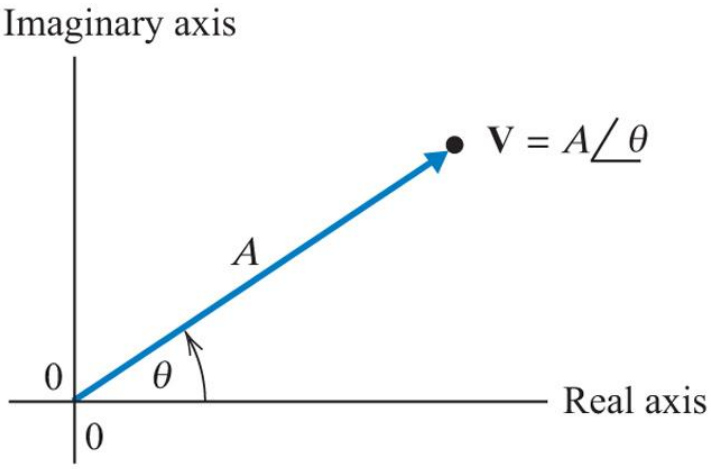
$j\omega L$

We call this as “Phasor”

Complex Plane

- Complex number can be represented in the plane





Phasor Form : Amplitude + phase angle

Phasor Utilizing Euler's Formula

- Euler's Formula

$$e^{j\phi} = \cos\phi + j\sin\phi$$

- Then

$$A e^{j\phi} = A \angle \phi$$

Phasor Form of Sinusoidal Voltage or Current

- Phasor is used for representing sinusoidal signal.

$$A\cos(\omega t+\theta) \quad \leftrightarrow \quad A\angle\theta$$

- ω is omitted? Why?
- Will use bold uppercase for phasor terms for clarity.
- Example 10.3-1. $i_1(t) = 120\cos(400t + 60^o)$ & $i2(t) = 100\sin(400t - 75^o)$

Arithmetic Using Phasor

- Addition $$\mathbf{V}_1 + \mathbf{V}_2 = (a + j\,b) + (c + j\,d) = (a + c) + j\,(b + d)$$

- Subtraction $$\mathbf{V}_1 - \mathbf{V}_2 = (a + j\,b) - (c + j\,d) = (a - c) + j\,(b - d)$$

- Multiplication & Division $$\mathbf{V}_1 \cdot \mathbf{V}_2 = \left(E \angle \theta\right)\left(F \angle \phi\right) = EF \angle (\theta + \phi) \text{ and } \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{A \angle \theta}{B \angle \phi} = \frac{A}{B} \angle (\theta - \phi)$$

- Conjugate $$\begin{aligned} \mathbf{V}_1{}^* &= (a + jb)^* = a - jb \\ &= \left(E \angle \theta\right)^* = E \angle -\theta \end{aligned}$$

Example 10.3-3

- $\mathbf{V}_1 = -1.796 + j3.852 = 4.25\angle 115^o$
- $\mathbf{V}_2 = -4+j3 = 5\angle 143^o$
- Find $\mathbf{V}_1+\mathbf{V}_2$, $\mathbf{V}_1 \cdot \mathbf{V}_2$, $\mathbf{V}_1/\mathbf{V}_2$

Relationship b/w V(ω) and v(t)

- Consider

$$v(t) = A\cos(\omega t + \theta) \text{ V and } \mathbf{V}(\omega) = A\angle\theta = Ae^{j\theta} \text{ V}$$

- Then,

$$v(t) = \mathrm{Re}\{\mathbf{V}(\omega)\, e^{j\omega t}\}$$

KVL & KCL in a Circuit

- Applying KVL

$$v(t) = \mathrm{Re}\{\mathbf{V}(\omega)\, e^{j\omega t}\}$$

$$0 = \sum_i v_i(t)$$

$$0 = \sum_i \mathrm{Re}\{\mathbf{V}_i(\omega)\, e^{j\omega t}\} = \mathrm{Re}\left\{ e^{j\omega t} \sum_i \mathbf{V}_i(\omega) \right\}$$

t = 0

$$0 = \mathrm{Re}\left\{ \sum_i \mathbf{V}_i(\omega) \right\}$$

t =

$$0 = \mathrm{Re}\left\{ -j \sum_i \mathbf{V}_i(\omega) \right\} = \mathrm{Im}\left\{ \sum_i \mathbf{V}_i(\omega) \right\}$$

0 = $\sum_i \mathbf{V}_i(\omega)$ **:** $\mathbf{V}_i(\omega)$ also satisfy KVL

Example 10.3-4

- $v_s(t) = 25 \cos(100t + 15^o)$ V
- $v_C(t) = 20 \cos(100t - 22^o)$ V
- $v_R(t) = ?$

