Chap. 10

Sinusoidal Steady-State Analysis (4)

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Analyzing an AC Circuit

- ✓ 10.9 Phasor Diagrams
- ✓ 10.10 Op Amps in AC Circuits
- ✓ 10.11 Complete Response

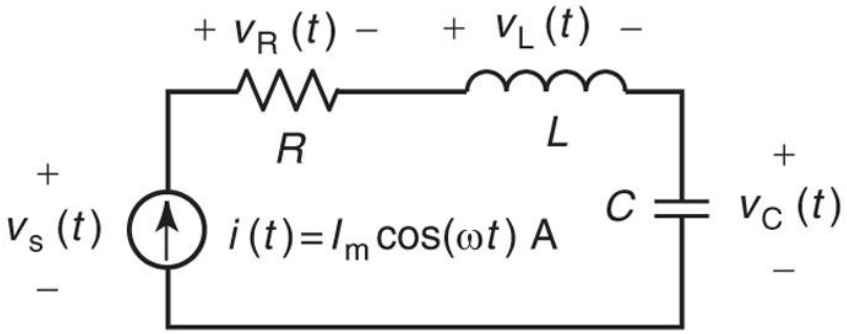
Phasor Diagram?

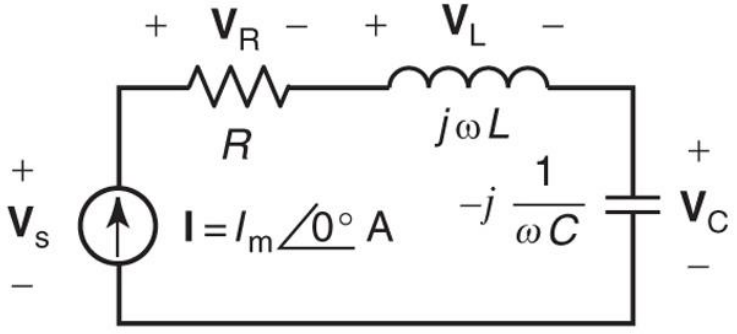
- A Phasor diagram is a graphical representation of phasors and their relationship on the complex plane.

- Ex)

Total Voltage of Series RLC Circuit

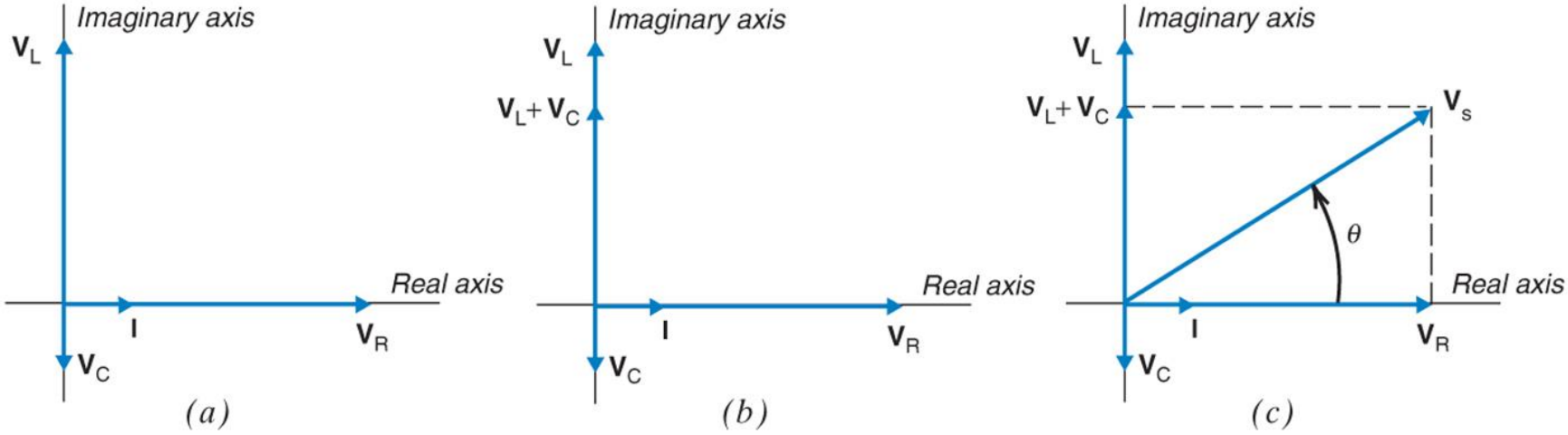
- Consider how V_S is determined.





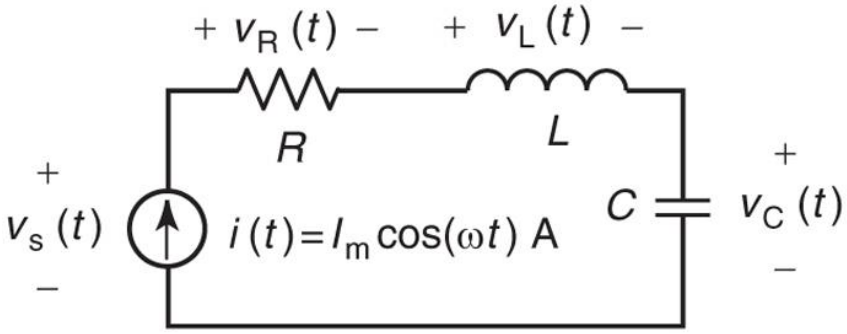
Phasor Diagrams for RLC Circuit

- Consider how V_S is determined.



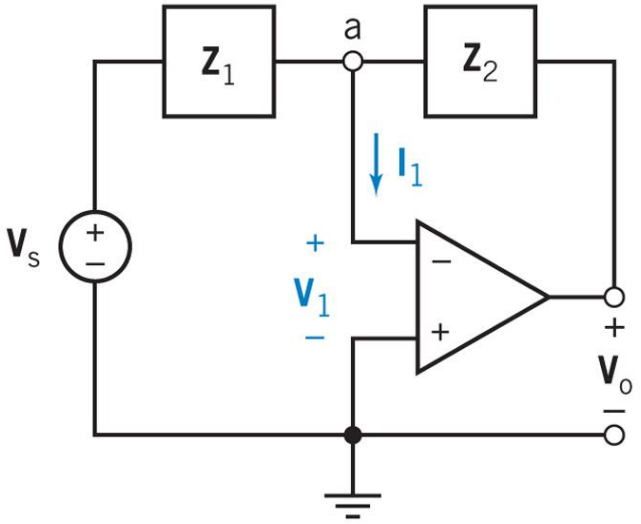
Example 10.9-1

- Consider the series RLC circuit when R = 80 Ω, L = 8 H, C = 5mF, and $i(t)$ = 0.25 cos (10t) A.

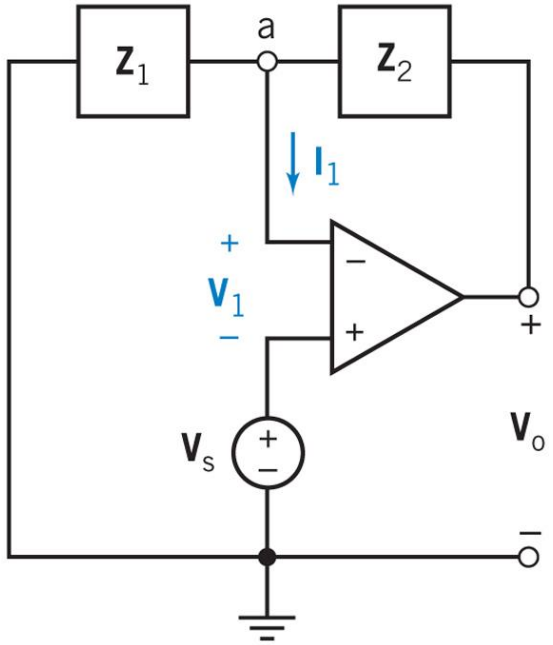


Op Amps in AC Circuits

- Consider the behavior of op-amps and associate RLC in the frequency domain with phasors.
- Frequently used structures are shown below.



Inverting Amplifier



Non-inverting Amplifier

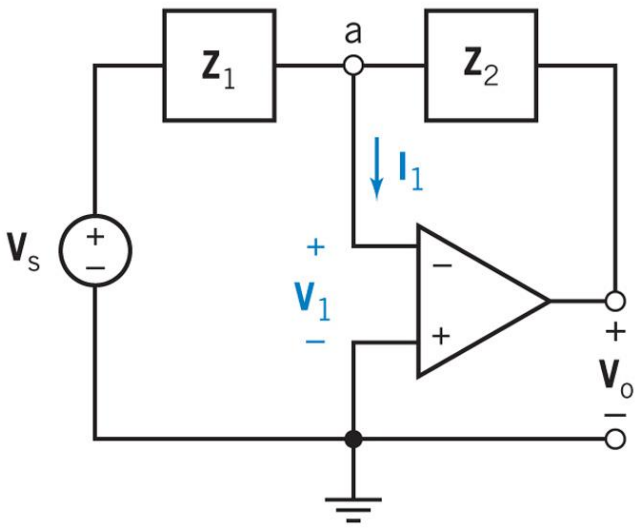
Inverting Amplifier

- Ratio of output-to-input voltage : V_o/V_s
- KCL at the node **a**.

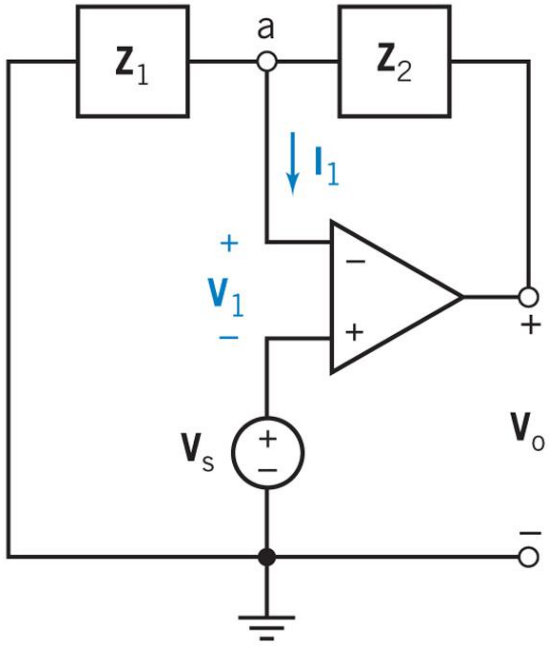
$$\frac{\mathbf{V}_s}{\mathbf{Z}_1} + \frac{\mathbf{V}_o}{\mathbf{Z}_2} = 0$$

- Then,

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1}$$



Noninverting Amplifier



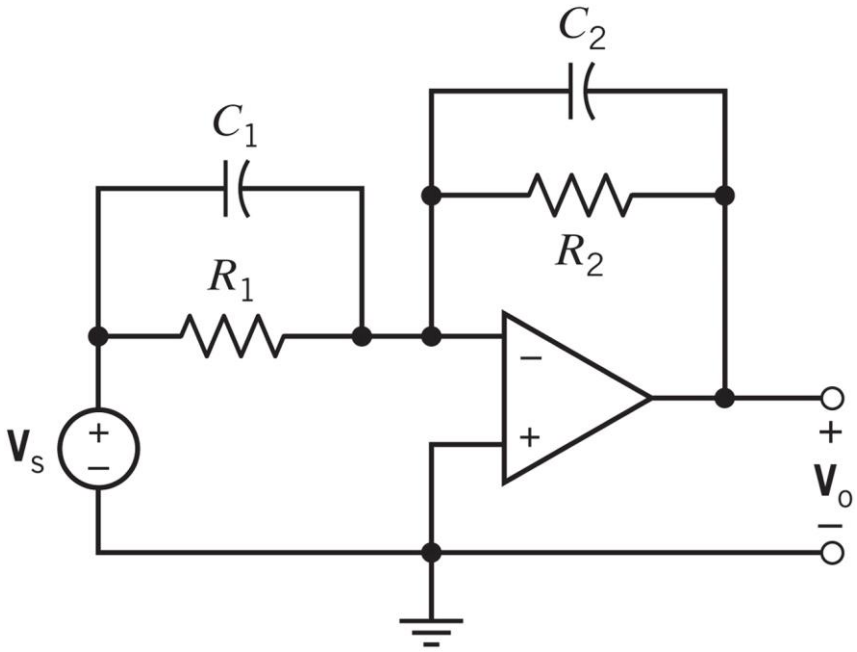
- Using KCL at the node a,

- Then,

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_1 + \mathbf{Z}_2}{\mathbf{Z}_1}$$

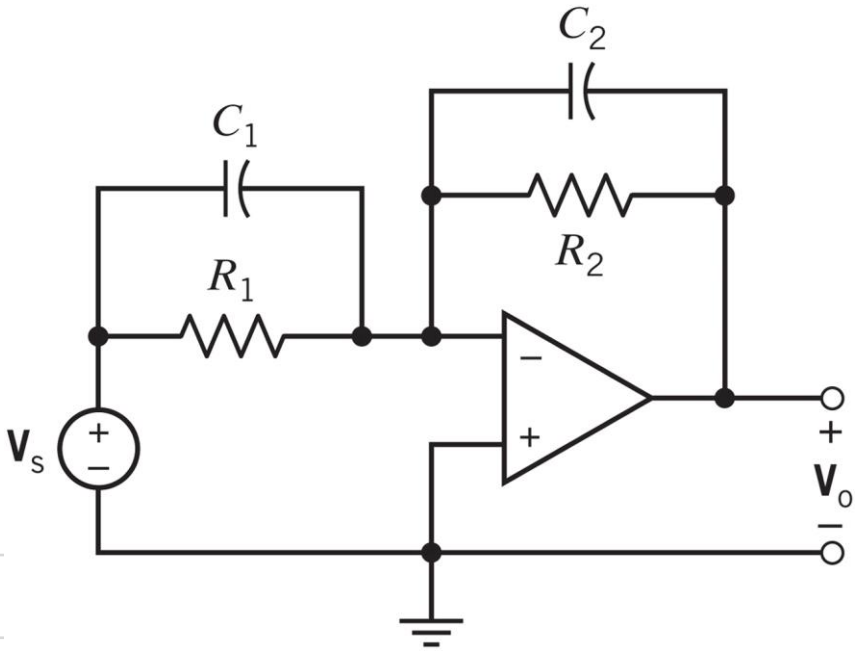
Example of Op-Amp Application

- $Z_1 = R_1 // \frac{1}{j\omega C_1}$
- $Z_2 = R_2 // \frac{1}{j\omega C_1}$
- $V_O/V_S = ?$



Example of 10.10-1

- Find V_O/V_S when R_1 = 1kΩ, R_2 = 10kΩ, C1 = 0, C2 = 0.1μF for ω = 1000 rad/s.



Revisit Natural & Forced Response

- In the circuit characterized by a differential equation, the complete response x(t) is given by

$$x = x_{\mathrm{n}} + x_{\mathrm{f}}$$

The term "(t)" is omitted

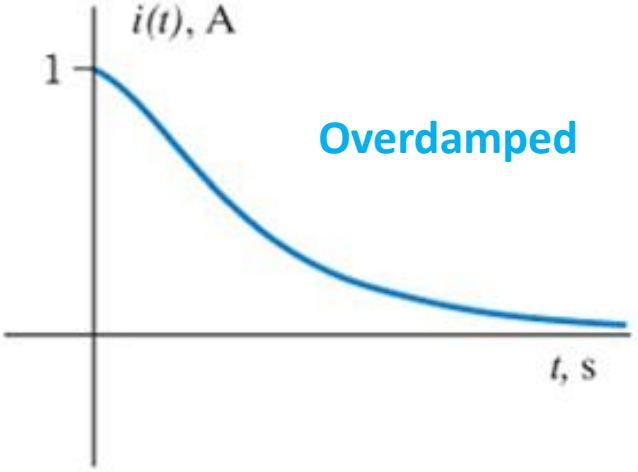
where x_{n} is natural response and x_{f} is forced response

- For example,

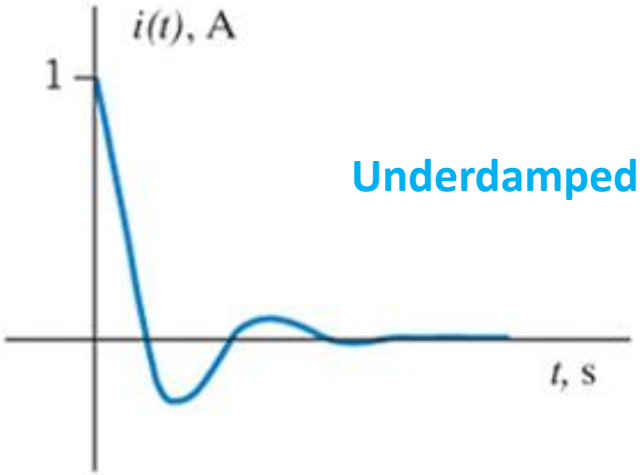
$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega^2{}_0 = f(\mathrm{t})$$

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega^2{}_0 = 0$$

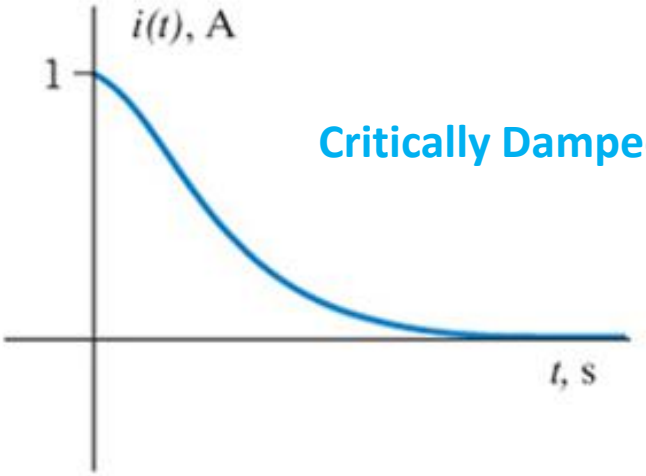
Possible Natural Responses



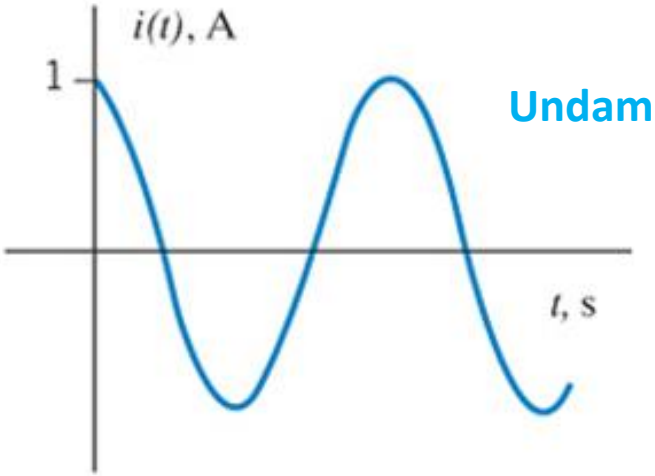
Overdamped



Underdamped



Critically Damped



Undamped

Questions

- We derived exp(st) that satisfy the differential equation, and s is the root of 2nd polynomial equation.
  - Why didn't we consider positive real root or
  - complex roots that not conjugate to each other.

$$v_n = A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$

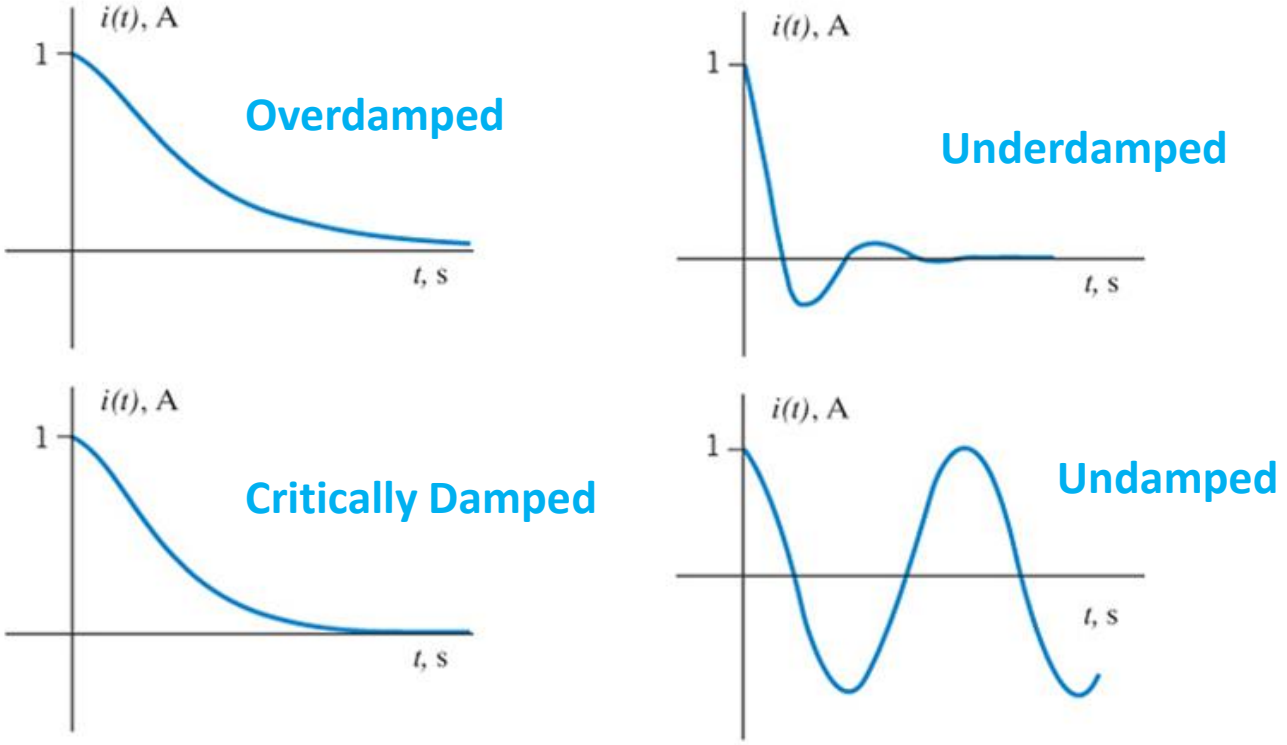
$$v_n = e^{-\alpha t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$$

$$\begin{aligned} v_n &= e^{-\alpha t}(A_1 \cos \omega_d t + jA_1 \sin \omega_d t + A_2 \cos \omega_d t - jA_2 \sin \omega_d t) \\ &= e^{-\alpha t}[(A_1 + A_2) \cos \omega_d t + j(A_1 - A_2) \sin \omega_d t] \end{aligned}$$

- Relations between sinusoidal, exponential, and constant?

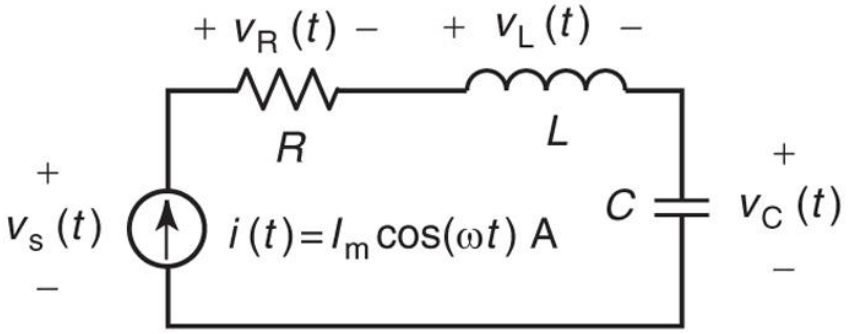
Possible Natural Response

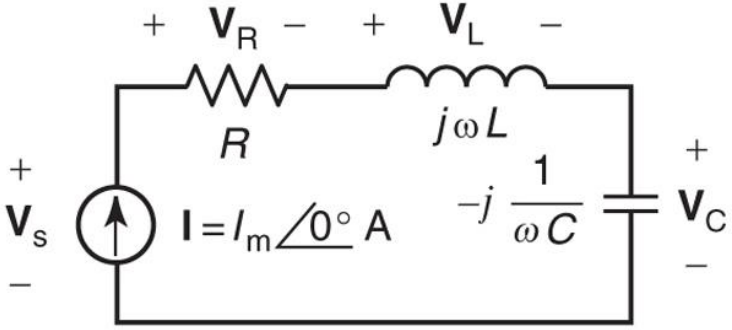
- Basically, it is exponential form, that is ∑exp(st) where s follows the following constraints.
  - If it has nonzero real part, it is negative
  - If it has only imaginary part, must be conjugate ➔ Sinusoidal.



Who is the Damper In RLC Circuit?

- Think of who “dissipates” energy





Complete Response

- Do you remember the definition of AC circuit?

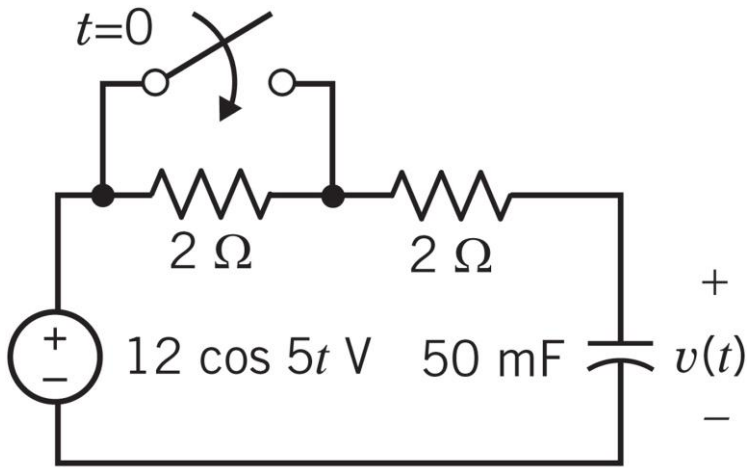
- How about other than sinusoidal input?
- Complete response?

Complete response = transient response + steady state response

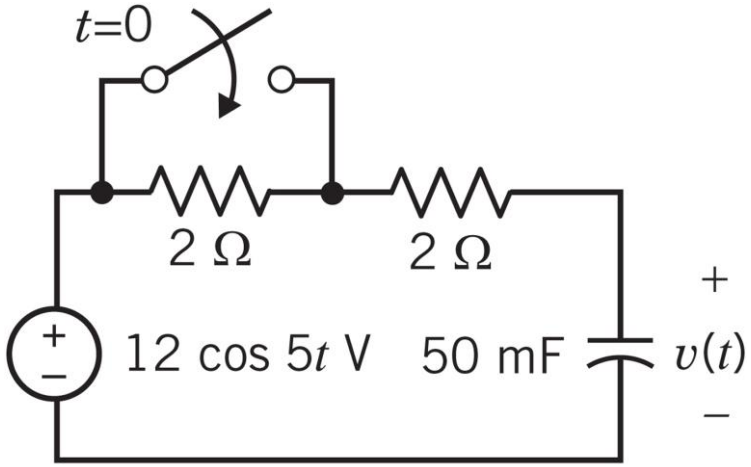
Complete response = natural response + forced response

Circuit with Fixed Sinusoidal Input

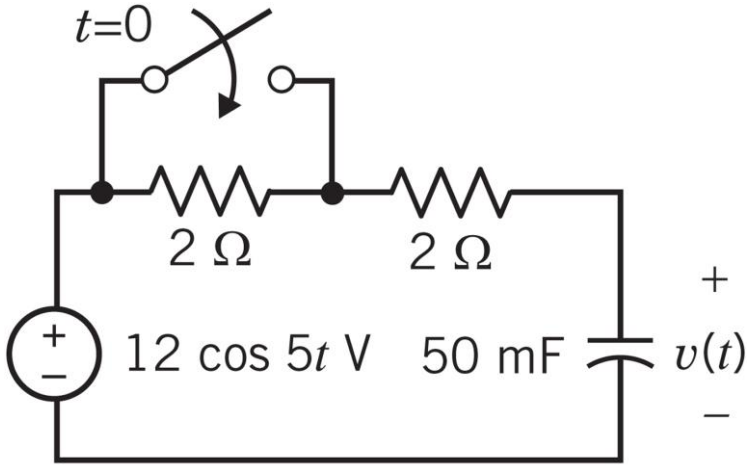
- What will happen at t = 0?



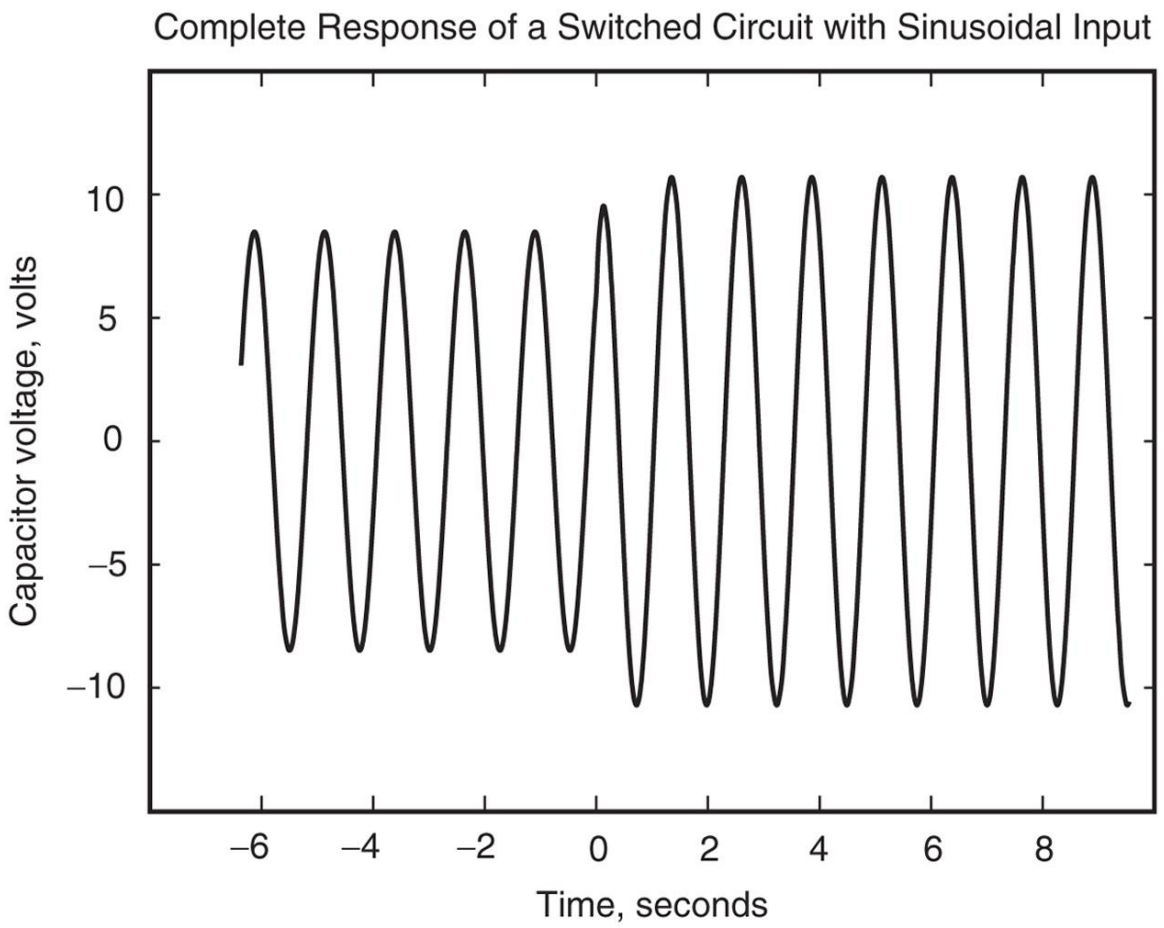
Complete Response



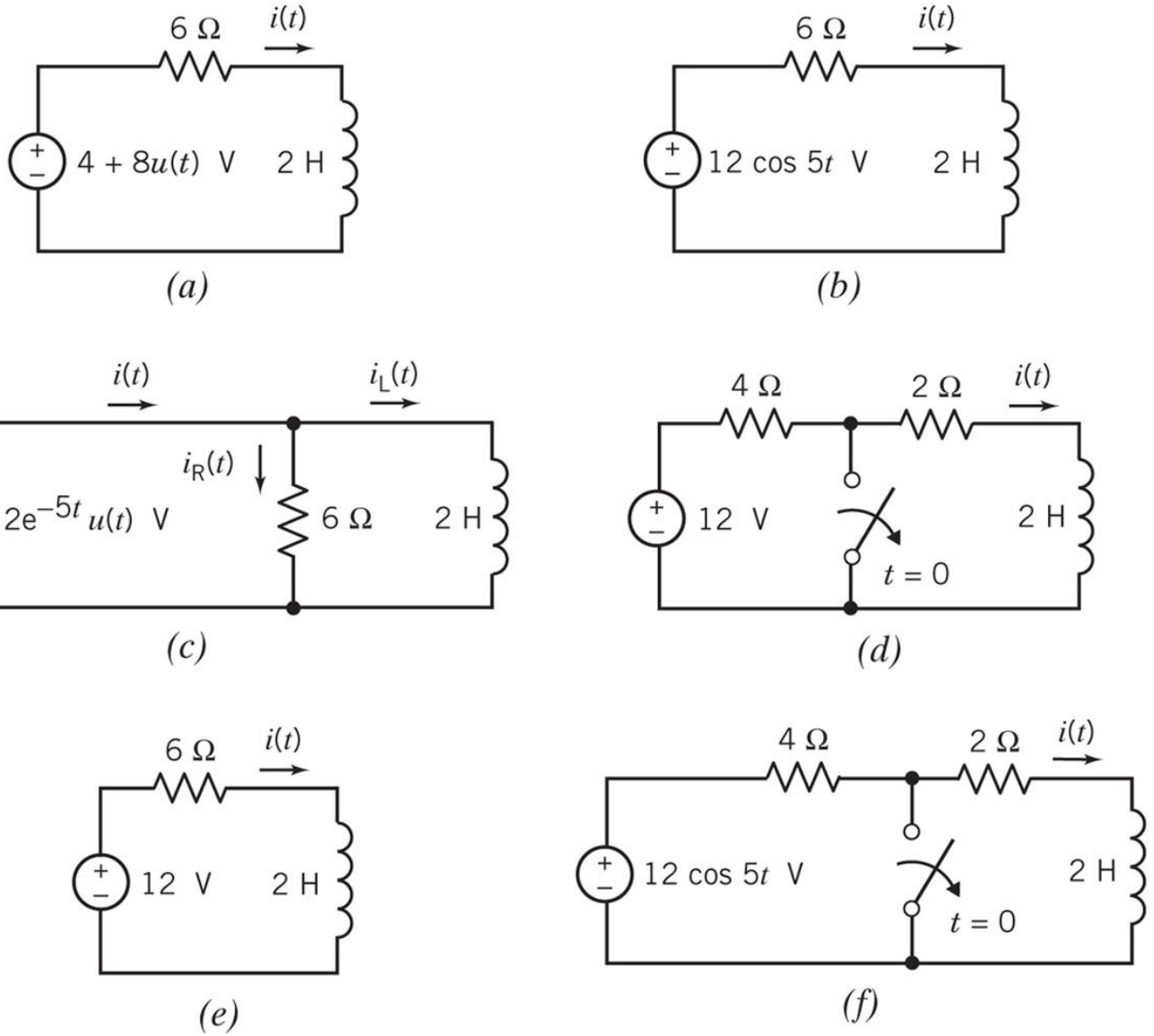
Complete Response



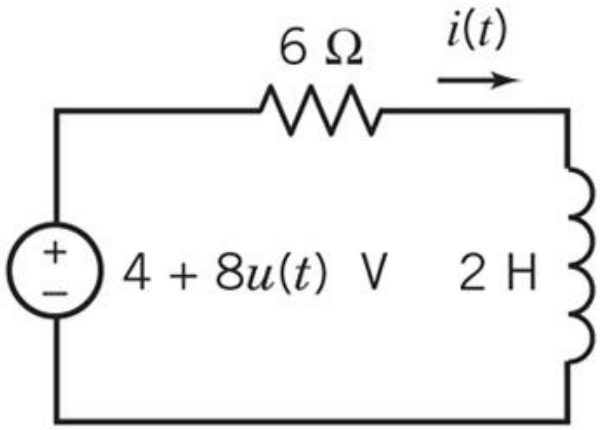
Complete Response



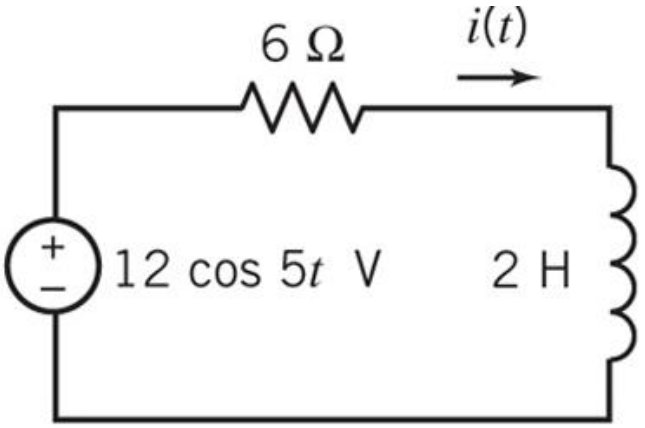
Example of 10.11-2



Example 10.11-2 (a)

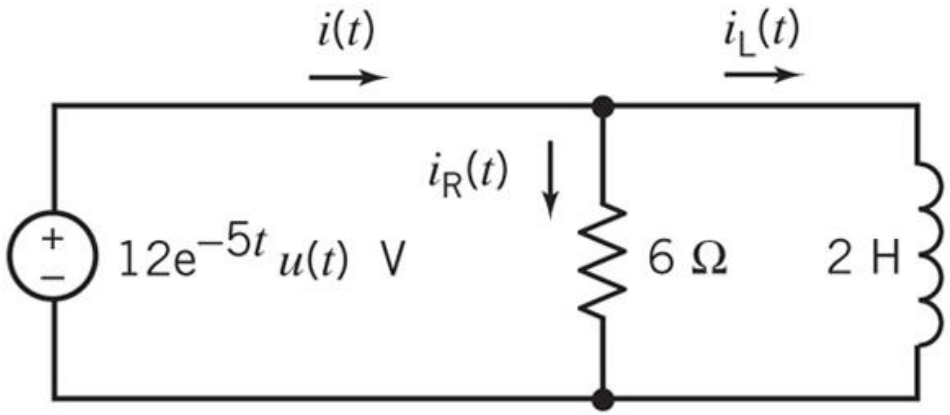


Example 10.11-2 (b)

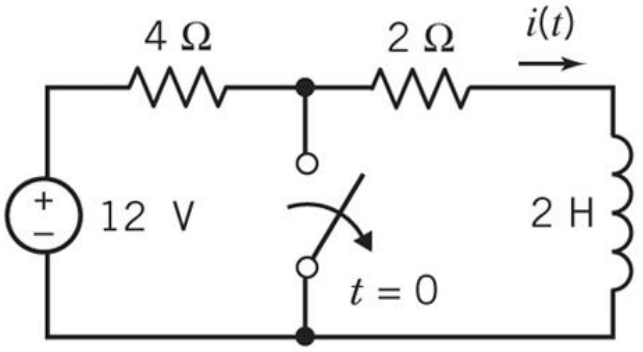


Example 10.11-2 (c)

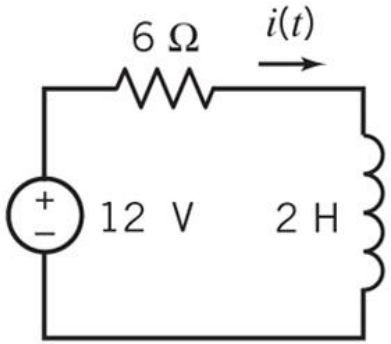
- Find i(t).



Example 10.11-2 (d)



Example 10.11-2 (e)



Example 10.11-2 (f)

