Chap. 11

AC Steady-State Power (1)

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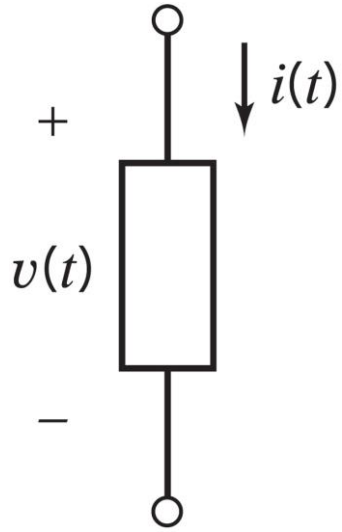
Various AC Power Terms

- ✓ 11.3 Instantaneous Power and Average Power
- ✓ 11.4 Effective Value of a Periodic Waveform
- ✓ 11.5 Complex Power

Instantaneous Power

- Power can be generated or absorbed in a circuit
- Instantaneous power delivered to the circuit shown below is :

$$p(t) = v(t)\, i(t)$$



Average Value of Periodic Function

- Suppose that v(t) is a periodic function having period T. That is,

$$v(t) = v(t+T)$$

- For a linear circuit, then i(t) is also periodic with period T, which is

$$i(t) = i(t+T)$$

- Then instantaneous power is

$$p(t) = v(t) \times i(t) = v(t+T) \times i(t+T) = p(t+T)$$

- The average power is

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$$

Average Power for Sinusoidal Voltage & Current

- Suppose that v(t) is sinusoidal that is

$$v(t) = V_m \cos(\omega t + \theta_V)$$

- For a linear circuit at steady state, the current will also be sinusoidal and will have the same frequency so

$$i(t) = I_m \cos(\omega t + \theta_I)$$

- The instantaneous power is

$$p(t) = v(t) \times i(t) = V_m I_m \cos(\omega t + \theta_V) \cos(\omega t + \theta_I)$$
$$= \frac{V_m I_m}{2} \{\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)\}$$

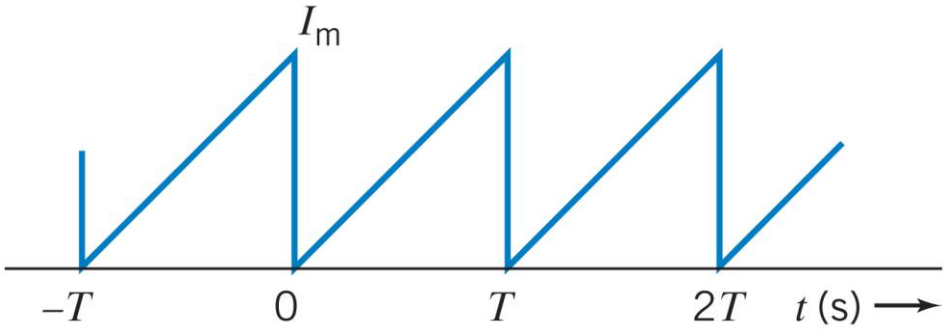
- The average power is

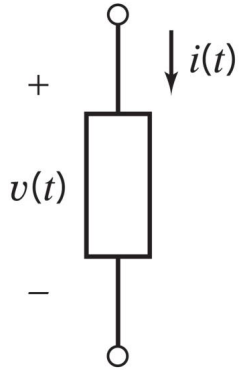
$$P = \frac{1}{T} \int_0^T p(t) dt$$

$$= \frac{V_m I_m}{2} \cos(\theta_V - \theta_I)$$

Example 11.3-1

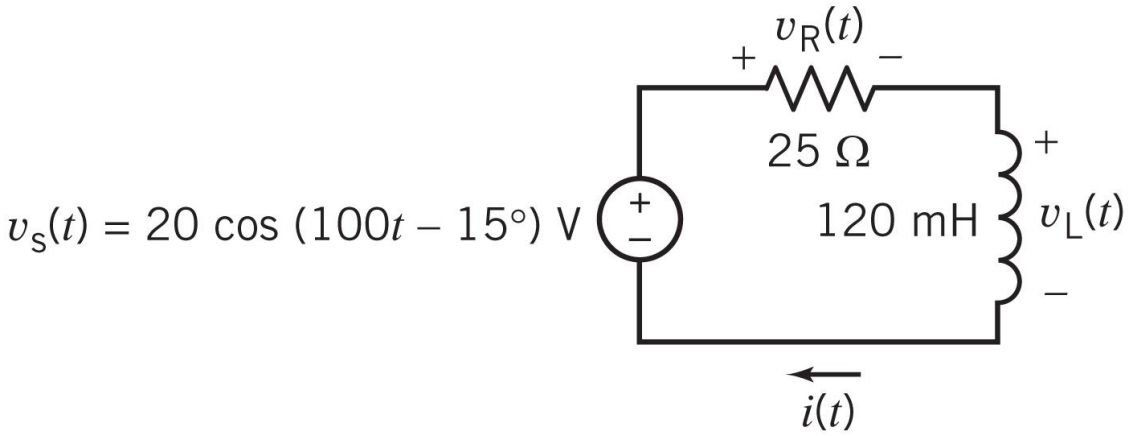
- Find the average power delivered to R when the current through R is like shown below.





Example 11.3-2

- Find the average power delivered to each device in the circuit at the steady state.



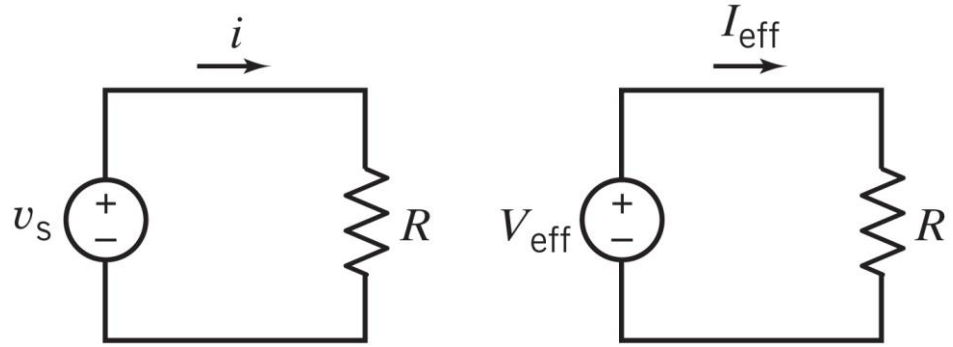
Average Power Delivered to L or C in AC Circuit

- Consider the i-v relation of L or C

- How do they response to sinusoidal signal?

Effective Value of a Periodic Current

- The goal of the effective value for varying voltage/current is to find a dc voltage that can deliver the same average power.



- The power delivered to the resistor R by a periodic current is

$$P = \frac{1}{T}\int_0^T i^2 R\, dt \qquad P = I_{\text{eff}}^2\, R$$

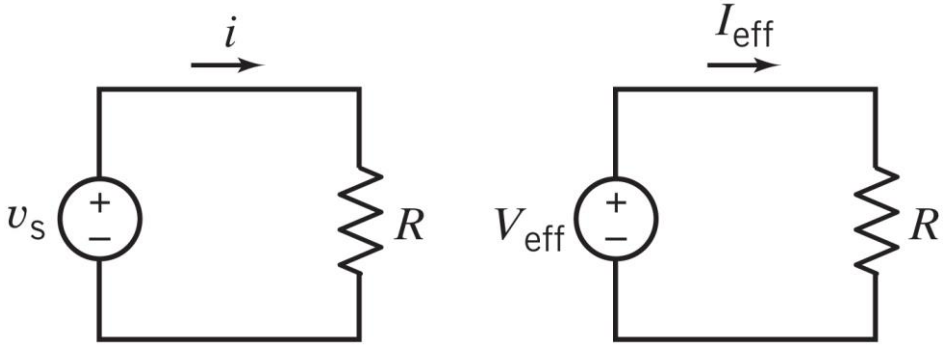
- Equalizing the above two,

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

➔ Called as root mean square current, I_{rms}

Effective Value of a Periodic Voltage

- The power can be expressed with Voltage as:



$$P = \frac{1}{T}\int_0^T \frac{v^2}{R}\,dt \qquad P = \frac{V_{eff}^2}{R}$$

- Equalizing the above two,

$$V_{rms}^2 = V_{eff}^2 = \frac{1}{T}\int_0^T v^2 dt$$

Root Mean Square (RMS) for Sinusoidal Voltage/Current

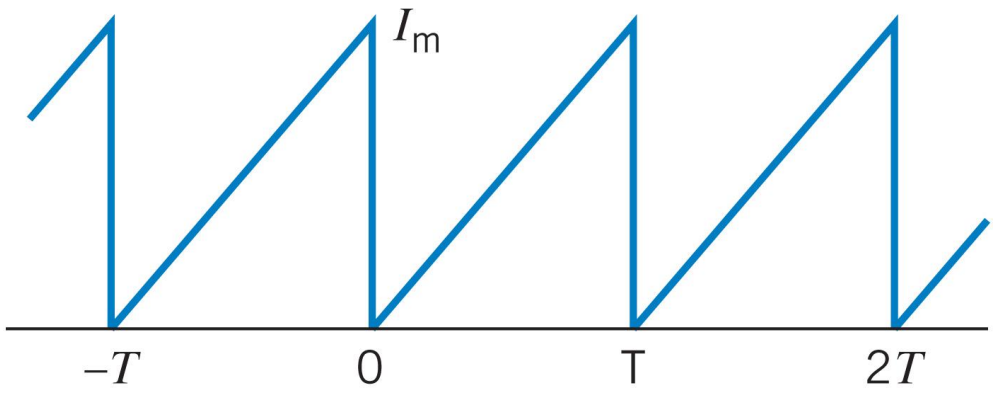
- When $i(t) = I_m \cos \omega t$, I_{rms} can be found as:

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_{\text{m}}^2 \cos^2 \omega t \, dt} = \sqrt{\frac{I_{\text{m}}^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t) \, dt} = \frac{I_{\text{m}}}{\sqrt{2}}$$

- How about voltage?

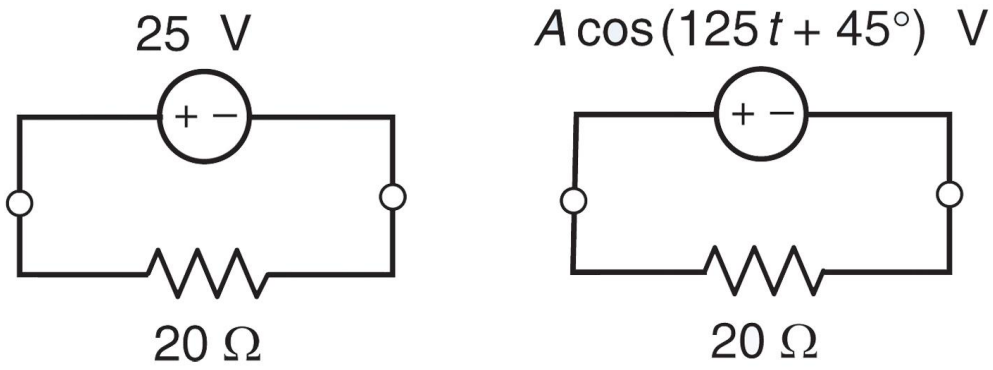
Example 11.4-1

- Find the effective value for the current shown:



Example 11.4-2

- The sources in the ac and dc circuits shown supply equal values of the average power. Determine the average power and the amplitude of the sinusoidal voltage.



Complex Power

- Suppose that a linear circuit with sinusoidal input is steady state.
- All the element voltages and currents will be sinusoidal, with the same frequency as the input ➔ analyzed thru phasor & impedances
- We can use them for power analysis as well.

$$\mathbf{I}(\omega) = I_{\mathrm{m}} \angle \theta_{\mathrm{I}} \quad \text{and} \quad \mathbf{V}(\omega) = V_{\mathrm{m}} \angle \theta_{\mathrm{V}}$$

- Complex Power :

$$\mathbf{S} = \frac{\mathbf{V}\mathbf{I}^*}{2} = \frac{(V_{\mathrm{m}} \angle \theta_{\mathrm{V}})\left(I_{\mathrm{m}} \angle -\theta_{\mathrm{I}}\right)}{2} = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} \angle \theta_{\mathrm{V}} - \theta_{\mathrm{I}}$$

- The magnitude of S is called **apparent power**.

$$|\mathbf{S}| = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2}$$

Average Power & Reactive Power

- Rewriting S with the rectangular term,

$$\mathbf{S} = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} \cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}}) + j \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} \sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$$

- We can represent complex power as

$$\mathbf{S} = P + jQ$$

- P is real part of S, which is the average power:

$$P = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} \cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$$

- Q is the imaginary part of S, which is the reactive power:

$$Q = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} \sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$$

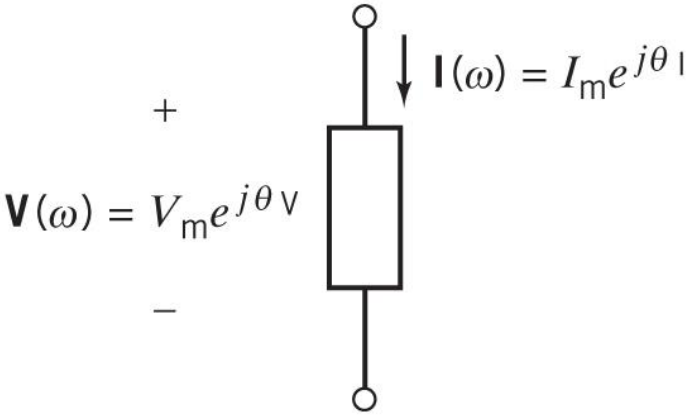
Frequency Domain Power Relationships

QUANTITY	RELATIONSHIP USING PEAK VALUES	RELATIONSHIP USING rms VALUES	UNITS
Element voltage, $v(t)$	$v(t) = V_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{v}})$	$v(t) = V_{\mathrm{rms}}\sqrt{2} \cos(\omega t + \theta_{\mathrm{V}})$	V
Element current, $i(t)$	$i(t) = I_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{I}})$	$i(t) = I_{\mathrm{rms}}\sqrt{2} \cos(\omega t + \theta_{\mathrm{I}})$	A
Complex power, $\mathbf{S}$	$\mathbf{S} = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2} \cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}}) + j\frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2} \sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	$\mathbf{S} = V_{\mathrm{rms}}I_{\mathrm{rms}} \cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}}) + jV_{\mathrm{rms}}I_{\mathrm{rms}} \sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	VA
Apparent power, $\lvert\mathbf{S}\rvert$	$\lvert\mathbf{S}\rvert = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2}$	$\lvert\mathbf{S}\rvert = V_{\mathrm{rms}}I_{\mathrm{rms}}$	VA
Average power, P	$P = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2}\cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	$P = V_{\mathrm{rms}}I_{\mathrm{rms}}\cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	W
Reactive power, Q	$Q = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2}\sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	$Q = V_{\mathrm{rms}}I_{\mathrm{rms}}\sin(\theta_{\mathrm{V}} - \theta_{\mathrm{I}})$	VAR

Relationship between Impedance & Complex Power

- The impedance can be represented as:

$$\mathbf{Z}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)} = \frac{V_m \angle\theta_V}{I_m \angle\theta_I} = \frac{V_m}{I_m} \angle\theta_V - \theta_I$$



- In the rectangular term,

$$\mathbf{Z}(\omega) = \frac{V_m}{I_m}\cos(\theta_V - \theta_I) + j\frac{V_m}{I_m}\sin(\theta_V - \theta_I)$$

Resistance Reactance

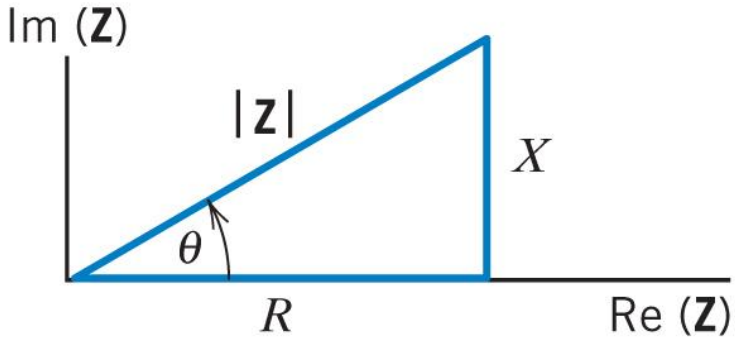
- Complex power can be expressed as

$$\mathbf{S} = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) + j\frac{V_m I_m}{2}\sin(\theta_V - \theta_I)$$

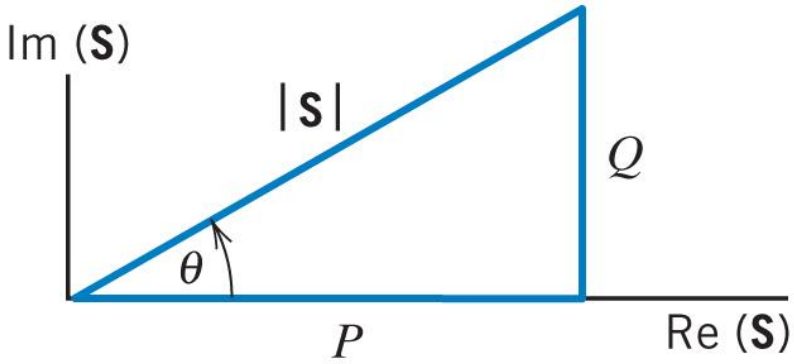
$$= \frac{I_m^2}{2}\mathrm{Re}(Z) + j\frac{I_m^2}{2}\mathrm{Im}(Z)$$

Average power Reactive Power

Impedance/Complex Power in Complex Plane



(a)



(b)