Chap. 11

AC Steady-State Power (2)

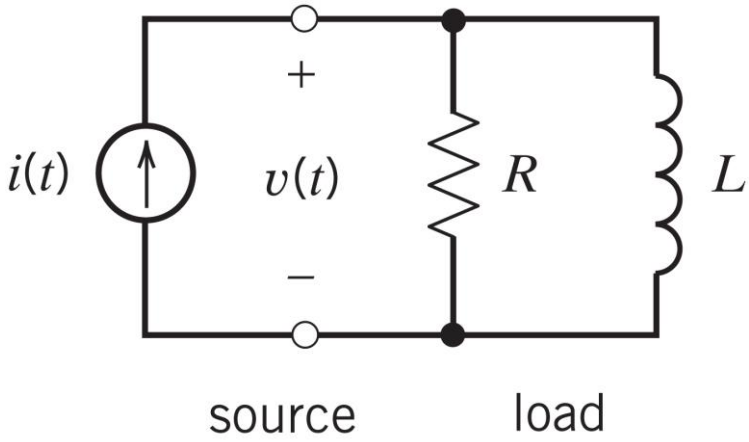
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AC Power Transfer

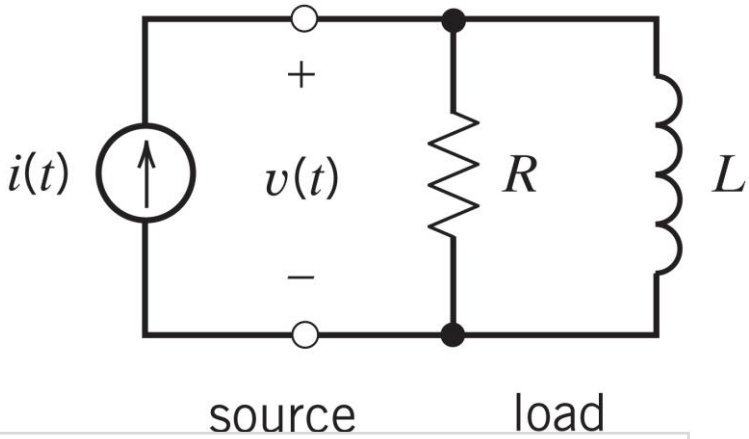
- ✓ 11.5 Complex Power
- ✓ 11.6 Power factor
- ✓ 11.8 Maximum Power Transfer Theorem

Example 11.5-1



- Suppose that i(t) = 1.25 cos (5t – 15°).
- Find the followings :
  1) Complex power delivered by the source to the load when R = 20 Ω and L = 3H.
  2) Values of R & L when source delivers 11.72 + j11.72 VA to the load

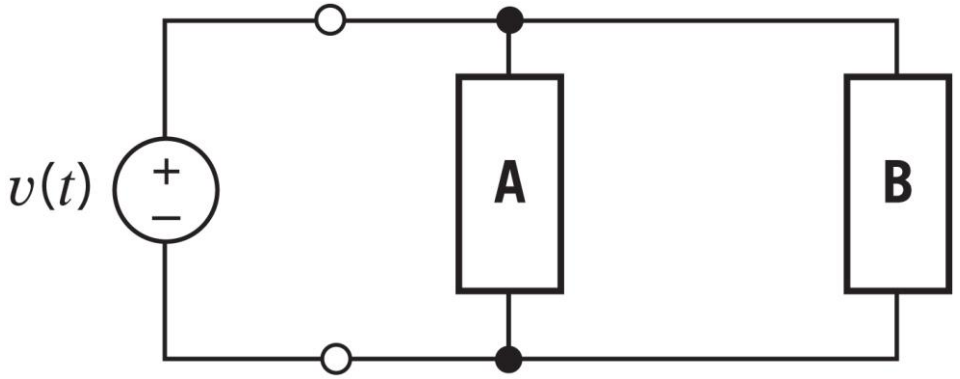
Example 11.5-1



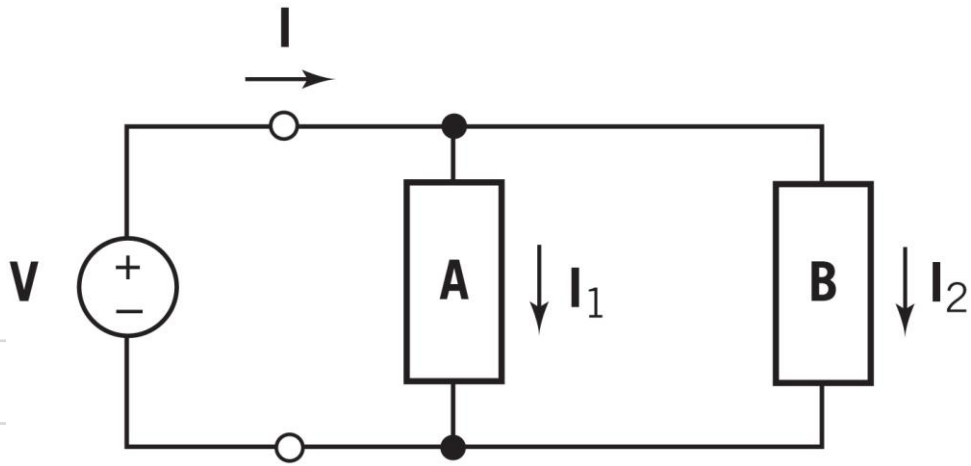
- Suppose that i(t) = 1.25 cos (5t – 15°)

Example 11.5-2

- **Suppose that :**
  - v(t) = 24 cos(5t+ 30°)
  - Complex power that load A receives S_A = 9.216 + j 6.912 VA
  - Impedance of load B, $Z_B = 42.426\angle 45^\circ\ \Omega$
- **Determine the followings :**
  1) Complex power delivered by source to the parallel load
  2) Equivalent impedance of the parallel load



Example 11.5-2



Conservation of Complex Power

- Complex power is conserved:

$$\sum_{\substack{all\\elements}} \frac{\mathbf{V}_k \mathbf{I}_k^*}{2} = 0$$

where V_k and I_k must adhere to the **passive convention** so that $V_k I_k^*/2$ is he complex power received ➔ $V_k I_k^*/2$ of sources supplying power is negative.

- Alternative expression :

NOT adheres to the passive convention

$$\sum_{sources} \frac{\mathbf{V}_k \mathbf{I}_k^*}{2} = \sum_{\substack{other\\elements}} \frac{\mathbf{V}_k \mathbf{I}_k^*}{2}$$

Adheres to the passive convention

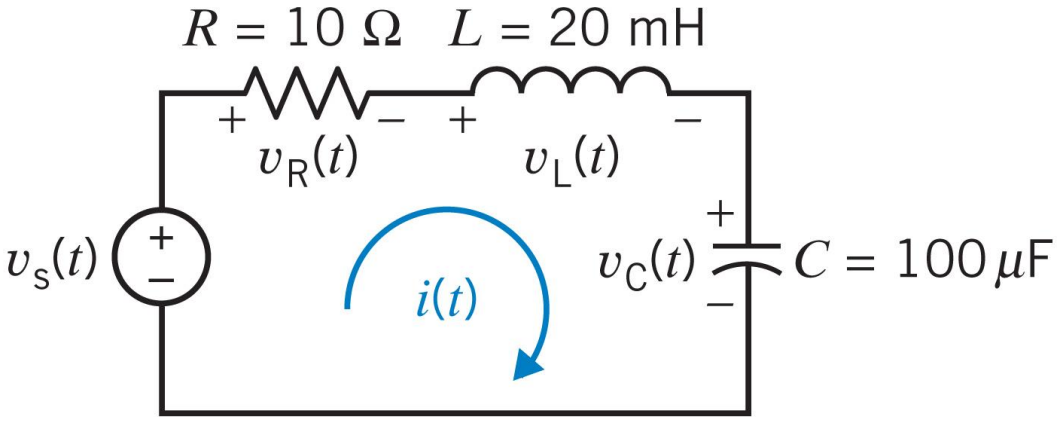
Conservation of Average/Reactive Power

- Conservation of the complex power implies that

$$\sum_{all\ Elements} Re\left(\frac{V_k I_k^*}{2}\right) = 0 \qquad \sum_{all\ Elements} Im\left(\frac{V_k I_k^*}{2}\right) = 0$$

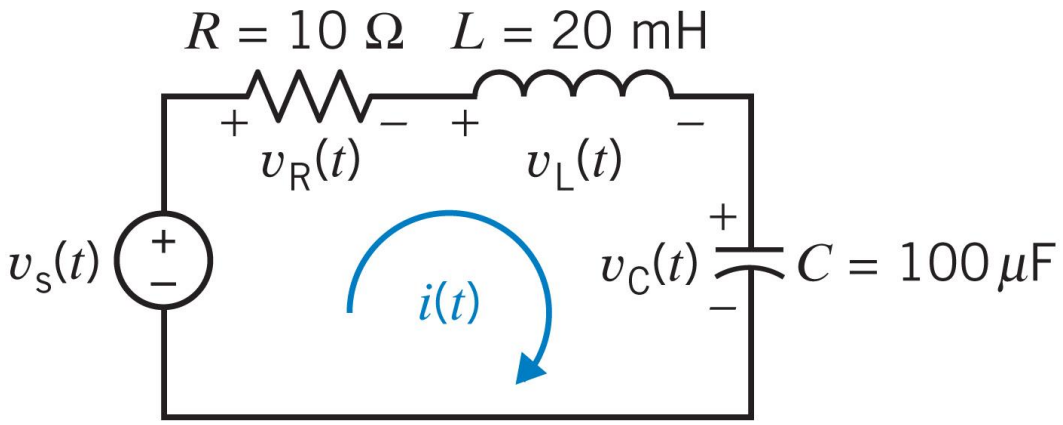
- In other words, average power and reactive power are both conserved.

Example 11.5-3



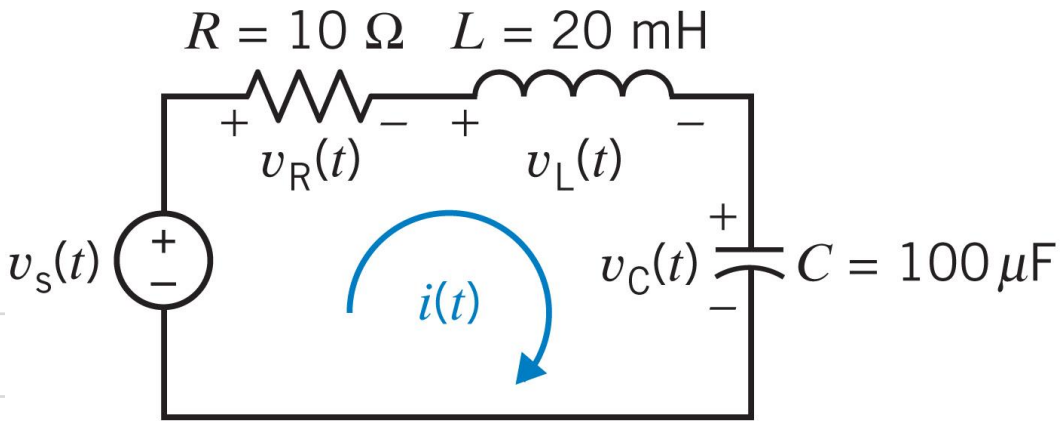
- Verify that complex power is conserved in the circuit
- v_s = 100 cos (1000t) V.

Example 11.5-3

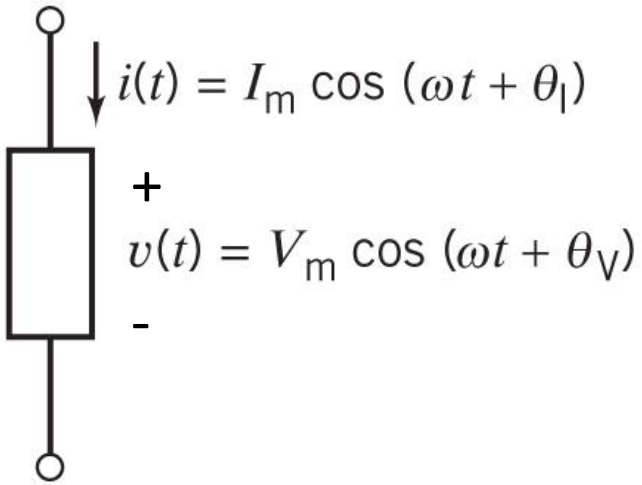


Example 11.5-4

- How about average power?



Power Factor



- Recall that the average power absorbed by the shown element:

and the apparent power is

- The ratio of the average power to the apparent power is called **power factor (pf)**, which means

power factor, pf = $\cos(\theta_V - \theta_I)$

- The average power is expressed as

$$P = \frac{V_m I_m}{2} pf$$

Leading & Lagging in Power Factor

- Cosine function is even function, which means

$$\cos(\theta_V - \theta_I) = \cos(\theta_I - \theta_V)$$

- pf = 0.8? ➔You cannot determine $(\theta_V - \theta_I)$ uniquely. As $\cos(36.87°) = \cos(-36.87°)$, $\theta_V - \theta_I = 36.87$ or -36.87.
- Thus, pf is labelled with **lagging** or **leading**.

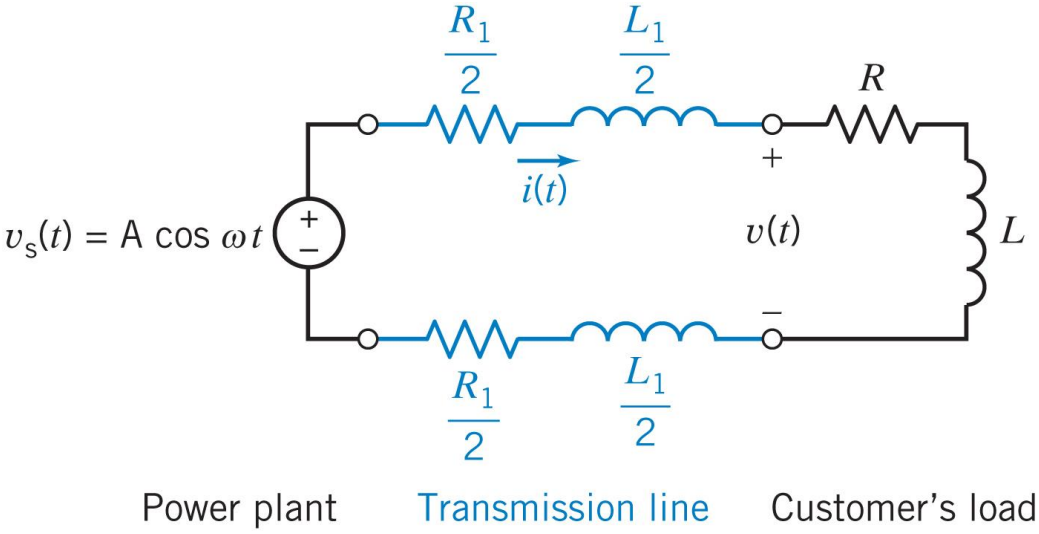
$$\theta_V - \theta_I > 0 : \text{Lagging}$$

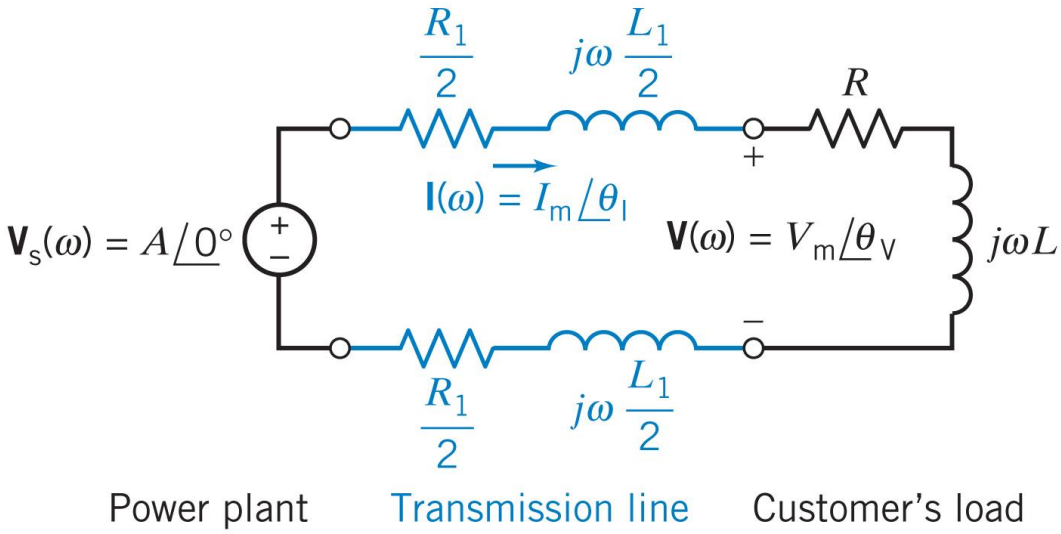
$$\theta_V - \theta_I < 0 : \text{Leading}$$

- For example, 0.8 leading / 0.8 lagging.

Model for Power Plant Supplying a Customer Electric Load

- Model representation of time/frequency domain





- Power plant supplies:

  Power used by the customer + Power absorbed by transmission line

  ➔ Should be minimized

How to Minimize P_{LINE}?

- R_1, L_1, V_m, P : typically given.
- Impedance of line:

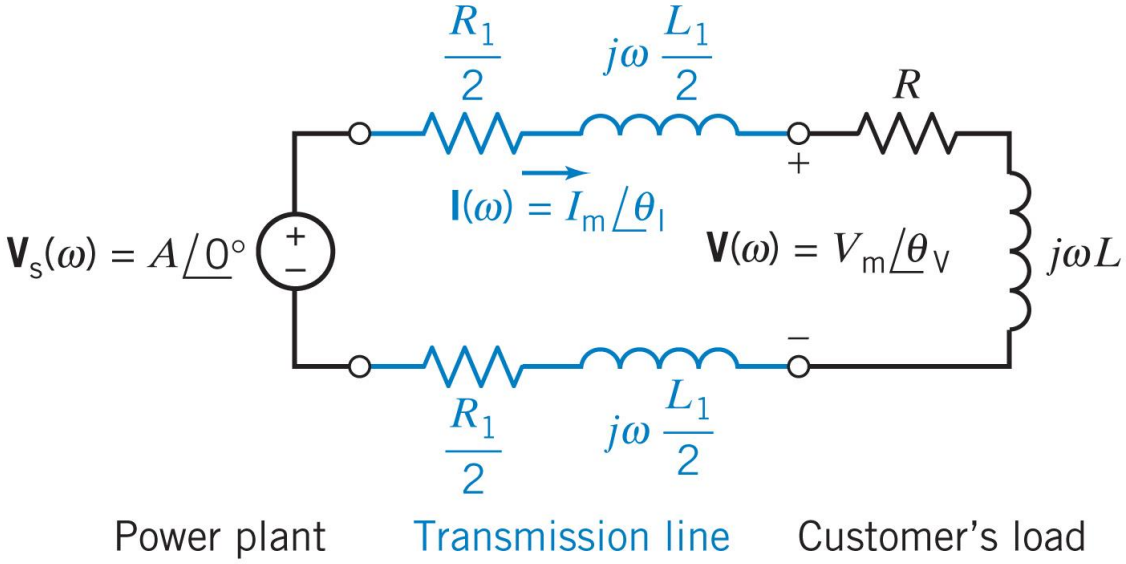
$$Z_{LINE}(\omega) = R_1 + j\omega L_1$$

- Average power absorbed by line:

$$P_{LINE} = \frac{I_m^2}{2} Re(Z_{LINE}) = \frac{I_m^2}{2} R_1$$

- P_{LINE} can be expressed with P, V_m as

$$P_{LINE} = 2\left(\frac{P}{V_m pf}\right)^2 R_1$$

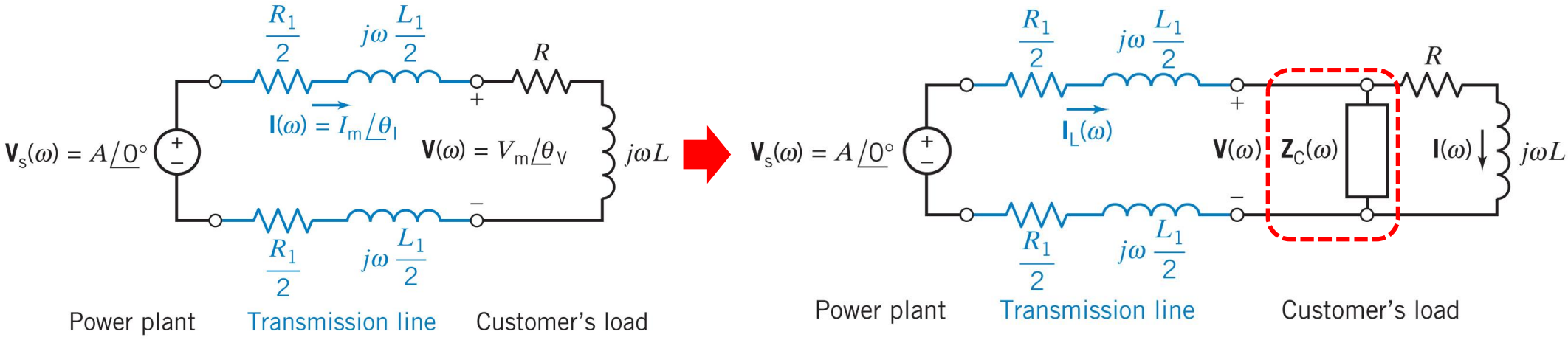


How to Control pf to Minimize P_{LINE}?

- pf = cos (θ_V-θ_I) should maximized to minimize PLINE while P, V_m, R_1 is fixed. How can you modify pf? What is the meaning of pf = 1 (the max.)?

$$P_{LINE} = 2\left(\frac{P}{V_m pf}\right)^2 R_1$$

- Compensating load impedance to modify pf as feasible.

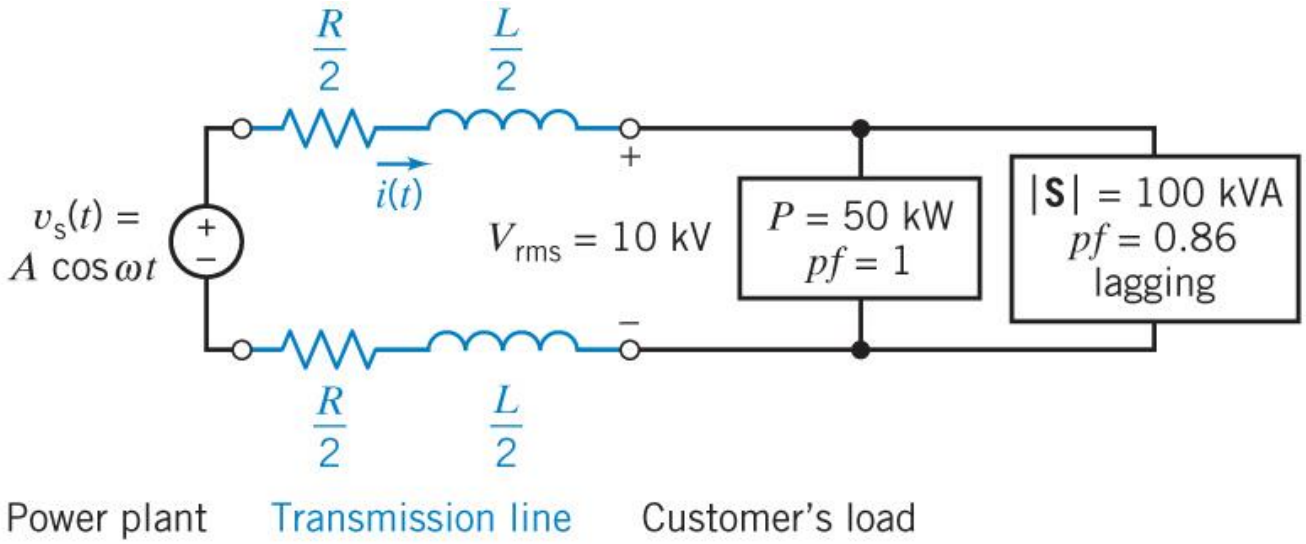


- Correcting power factor: **pf → pfc** (= corrected pf, pfc = cosθ_C)

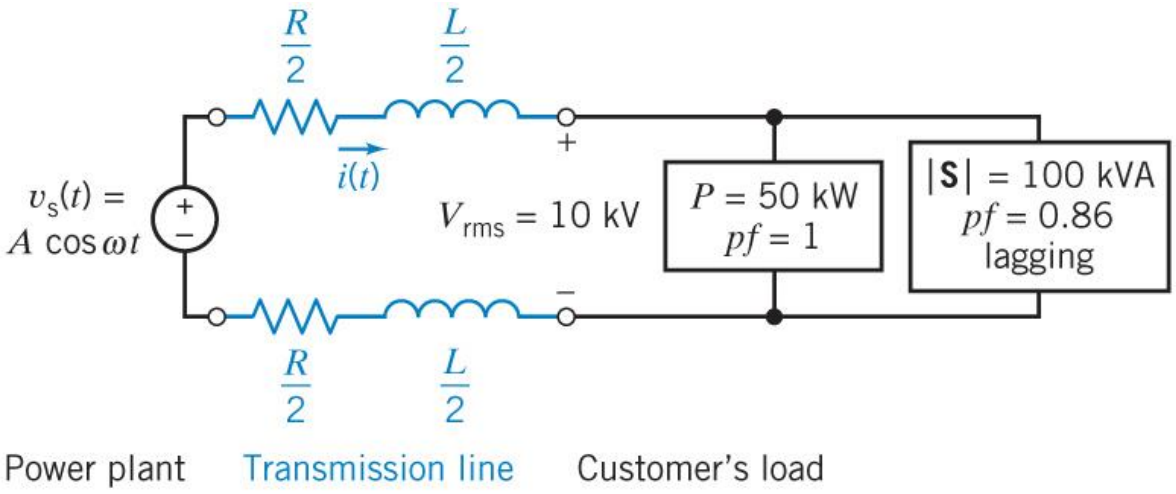
Example 11.6-1

- 1st load : 50kW resistive heating.
- 2nd load : Motors operating 0.86 lagging pf, The motors' load = 100 kVA.
- Power is supplied to the plant at 10,000 volts rms.

➔ Determine the total current flowing from the utility's lines to the customer's load and the its overall power factor.

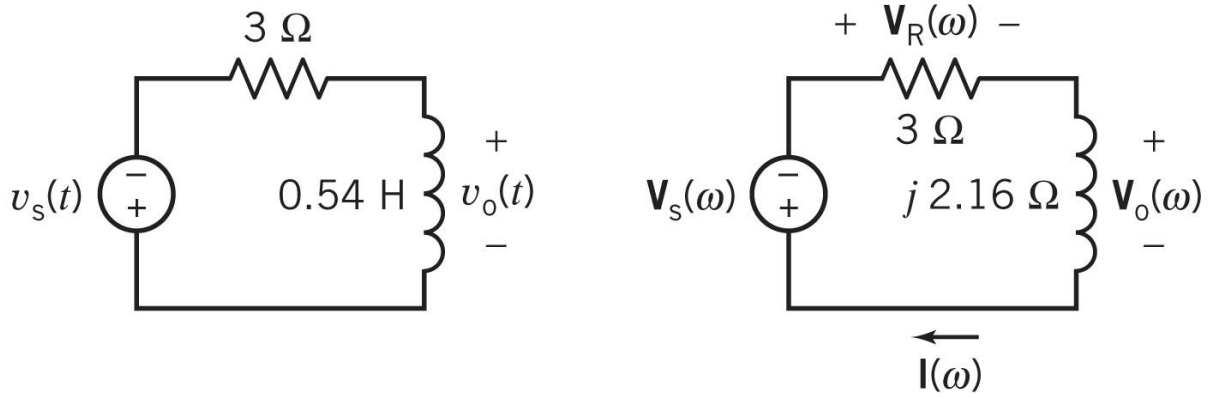


Example 11.6-1



Example 11.6-3

- Voltage source $v_s(t) = 7.28 \cos(4t + 77°)$ V.
- Voltage across the inductor $v_o(t) = 4.254 \cos(4t + 311°)$ V.



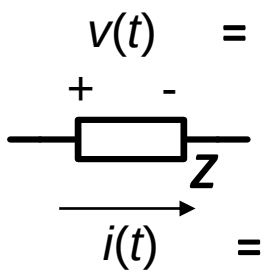
- Find the followings:

1) Average power supplied by the voltage source
2) Average power received by the resistor
3) Average power received by the inductor
4) Power factor of the impedance of RL series connection.

Example 11.6-3

Example 11.6-3

Review on AC Power



What if $\theta_V = \theta_I$? ➔ **Z = R**

What if $\theta_V \neq \theta_I$? ➔ **Z = R + jX**

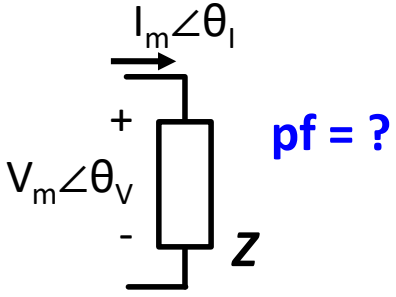
- Average power ➔ (Integral of instantaneous power over T) × **1/T**

$$P = \frac{1}{T}\int_0^T V_m I_m \cos(\omega t + \theta_V)\cos(\omega t + \theta_I)dt = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I)$$

- Complex power

$$S = \boldsymbol{V}\times\boldsymbol{I}^*/2 = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) + j\frac{V_m I_m}{2}\sin(\theta_V - \theta_I) = \mathbf{P} + j\mathbf{Q}$$

- Apparent Power? **=|S|** Power factor? **= P/|S| = cos($\theta_V - \theta_I$) = cos(∠S)**
- pf < 1 is due to $\theta_V \neq \theta_I$
  - ➔ Because Z has reactance! (i.e., Im(Z)≠0)
  - ➔ ∠Z = ∠(V/I) = **$\theta_V - \theta_I$**
  - ➔ pf = **cos(∠Z)**

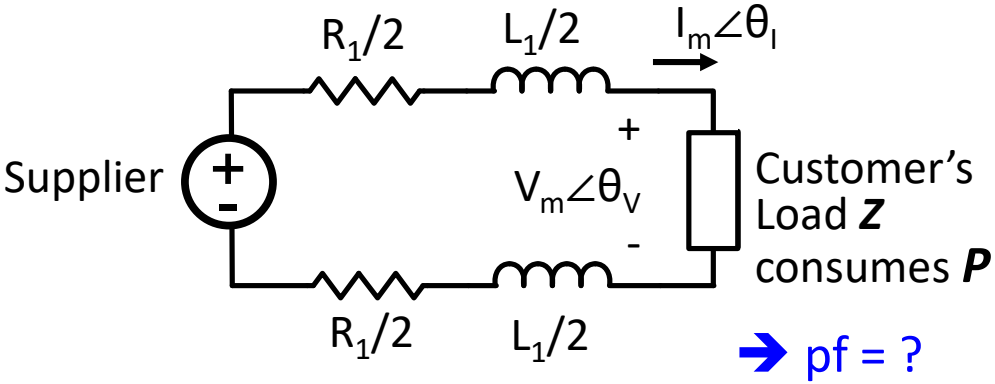


Motivation for Minimizing P_{LINE}

- Considering transmission line

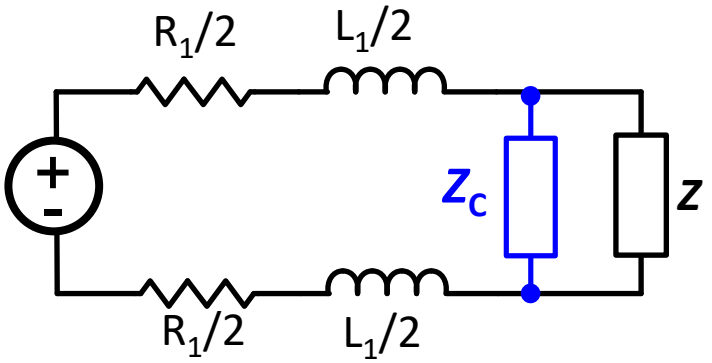
$$P_{LINE} = \frac{I_m^2}{2} Re(Z_{LINE}) = \frac{I_m^2}{2} R_1$$
$$= 2\left(\frac{P}{V_m pf}\right)^2 R_1$$

➔ pf = 1 for minimizing P_{LINE}

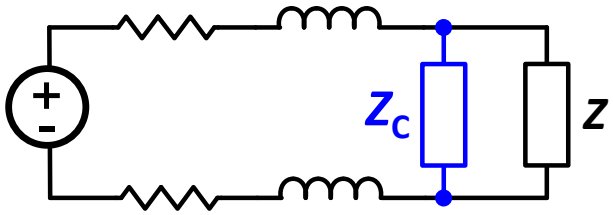


➔ pf = ?
= cos∠Z

- Correcting pf!
  - ➔How should we determine Z_C?
  - Before correction, Z ➔ pf = **cos∠Z**
  - After correction, $Z//Z_C$ ➔ pfc = **cos∠Z//Z_C**
- How Z_C should be to make pfc = target?
  - ➔ Let's look into this!



Compensating Impedance



How Z_C should be to make pfc = pfc_{target}?

- As we don't want additional load absorbs average power,

  Compensating impedance $Z_C = R_C + jX_C = jX_C$

- Impedance of the parallel combination $Z_P = Z//Zc$ is

$$Z_P = \frac{ZZ_C}{Z+Z_C} = R_P + jX_P = Z_P\angle\theta_P$$

- Power factor of the new combination is

$$pfc = \cos\theta_P = \cos\left(\tan^{-1}\frac{X_P}{R_P}\right)$$

- To calculate R_P and X_P :

$$Z_P = \frac{ZZ_C}{Z+ZC} = \frac{(R+jX)jX_C}{(R+jX)+jX_C} = \frac{RX_C^2+j[R^2X_C+(X_C+X)XX_C]}{R^2+(X+X_C)^2}$$

$$= \underbrace{\frac{RX_C^2}{R^2+(X+X_C)^2}}_{R_P} + j\underbrace{\frac{R^2X_C+(X_C+X)XX_C}{R^2+(X+X_C)^2}}_{X_P}$$

➔ How X_C should be to make pfc = pfc_{target}?

Deriving X_C

- Recall that

$$\text{pfc}_{\text{target}} = \cos\theta_P = \cos\left(\tan^{-1}\frac{X_P}{R_P}\right)$$

- Inserting determined X_P and R_P

$$\text{pfc}_{\text{target}} = \cos\left(\tan^{-1}\frac{R^2+(X_C+X)X}{RX_C}\right)$$

- Or we can we write

$$X_C = \frac{R^2+X^2}{R\ \tan\{\cos^{-1}(pfc_{target})\} - X}$$

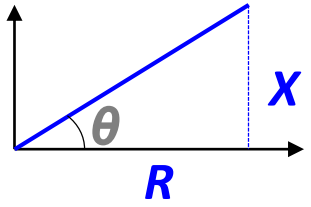
- Note that X_C may be positive or negative, depending on the target pfc and the original R and X of the load.
- $\tan\{\cos^{-1}(pfc_{target})\}$ will be positive if pfc is specified as lagging, and negative if pfc is specified as leading.

X_C with Inductive Customer Load

- When the customer load is inductive, that is R + jX (X>0), we will need a capacitive impedance Z_C.

$$\mathbf{Z}_C = \frac{-j}{\omega C} = jX_C$$

- With combining $X_C = \frac{R^2+X^2}{R\ \tan\{\cos^{-1}(pfc_{target})\} - X}$,

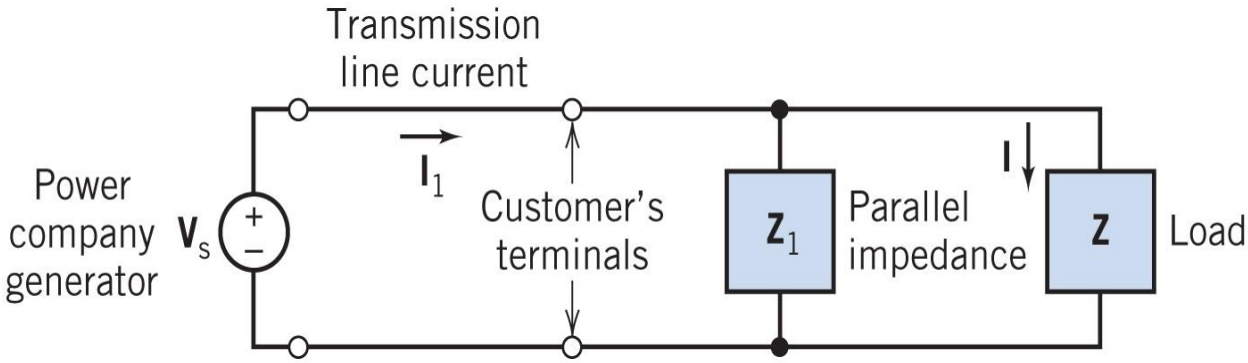


$$-1/\omega C = \frac{R^2+X^2}{R\ \tan\{\cos^{-1}(pfc_{target})\} - X}$$

$$\omega C = \frac{X - R\ \tan\{\cos^{-1}(pfc_{target})\}}{R^2+X^2} = \frac{X - R\ \tan\theta_P}{R^2+X^2} = \frac{R\tan\theta - R\tan\theta_P}{R^2+X^2}$$

$$\omega C = \frac{R}{R^2+X^2}(\tan\theta - \tan\theta_P)$$

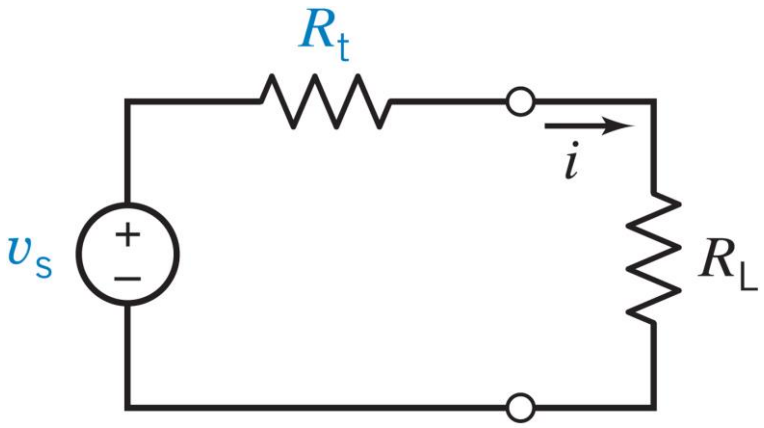
Example 11.6-2



- Load Z = 100 + j100 Ω.
- Source operates at the frequency ω = 377 rad/s.
- Find the parallel capacitance required to correct pf to (a) 0.95 lagging and (b) 1.0.

Maximum Power Transfer in Resistive Network

- What value RL should have to achieve the maximum power transfer condition, when R_t and v_s are given?
- Isn't it RL = 0? or infinite?
- Power absorbed in R_L is



$$p = \left(\frac{v_s}{R_L + R_t}\right)^2 R_L$$

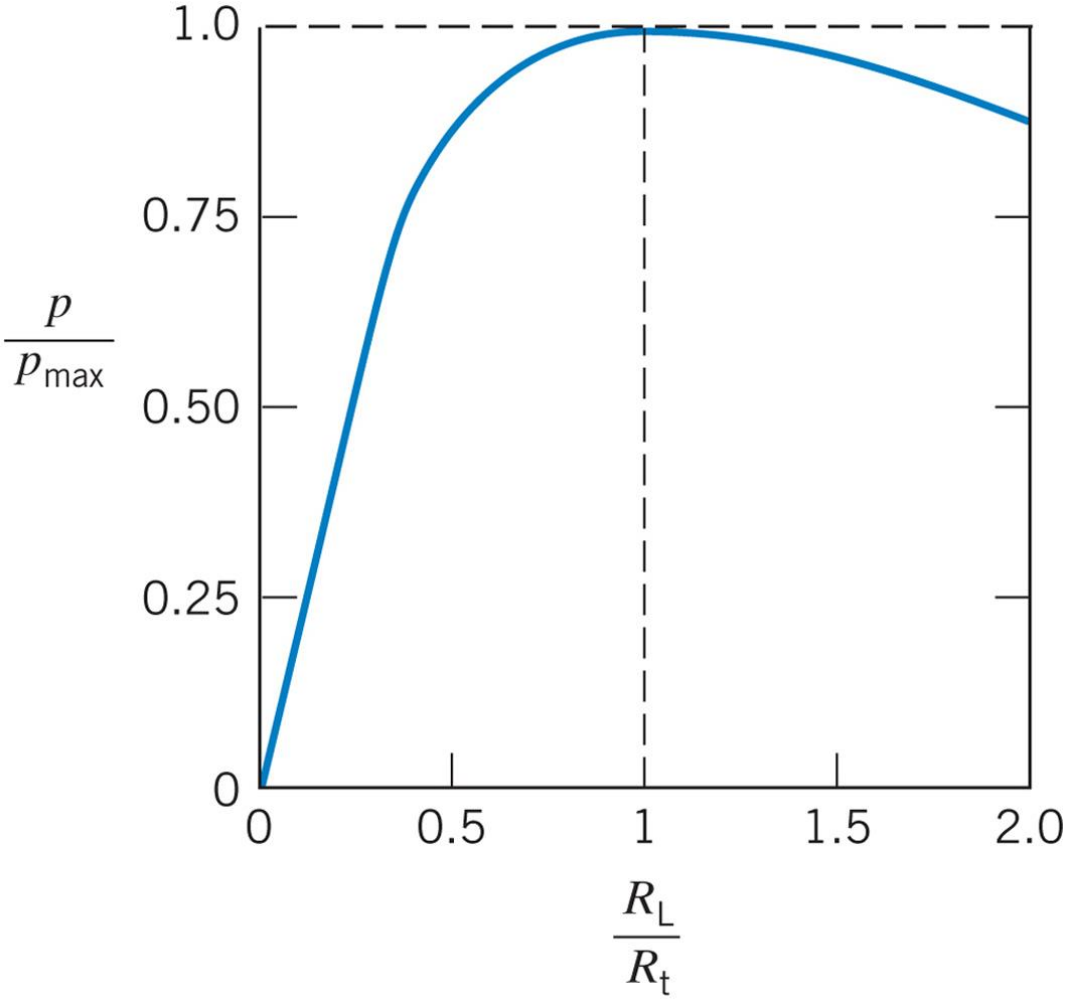
- Finding $\frac{dp}{dR_L}$,

$$\frac{dp}{dR_L} = v_s^2 \frac{(R_t + RL)^2 - 2(R_t + RL)R_L}{(R_L + Rt)^4}$$

- The derivative is zero when

$$(R_t + R_L)^2 - 2(R_t + R_L)R_L = 0$$

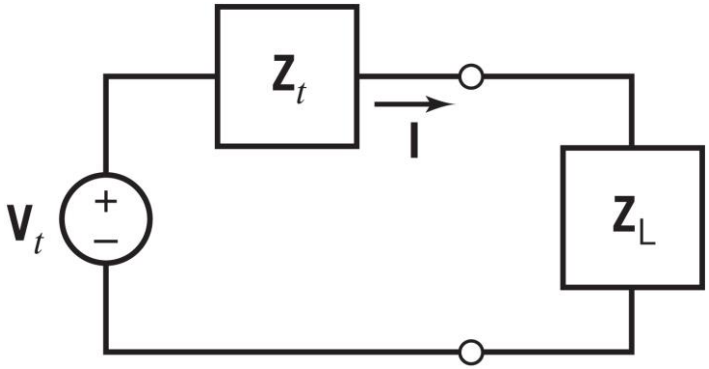
Power vs. R_L



Maximum Power Transfer in AC Circuit

- The average power delivered to the load is

$$P = \frac{I_m^2}{2} R_L$$



- The phasor current I is given by

$$I = \frac{V_t}{Z_t + Z_L} = \frac{V_t}{R_t + jX_t + R_L + jX_L}$$

- The average power is

$$P = \frac{I_m^2}{2} R_L = \frac{|V_t|^2 R_L/2}{(R_t + R_L)^2 + (X_t + X_L)^2}$$

- We wish to maximize P. First, we can pick $X_L = -X_t$, then $P = \frac{|Vt|^2 R_L/2}{(R_t + R_L)^2}$

➔ P is minimized when $R_L = R_t$ if we recall resistive case.

- Consequently, the maximum power transfer condition is

$$Z_L = R_t - jX_t = Z_t^*$$

Example 11.8-1

- Find the load impedance that transfers the maximum power to the load and determine the maximum power.

