Chap. 11

AC Steady-State Power (3)

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AC Power Transfer

- ✓ 11.7 Power Superposition
- ✓ 11.9 Coupled Inductors
- ✓ 11.10 Ideal Transformer

Review on Orthogonal Sinusoidal Functions

- Suppose that we integrate $f(t) = \cos\boldsymbol{m}t \times \cos\boldsymbol{n}t$ over one period of $f(t)$.

$$\int_0^{2\pi} \cos 2t \times \cos 3t \, dt = 0$$

$$\int_0^{\pi} \cos 2t \times \cos 4t \, dt = 0$$

$$\int_0^{\pi} \cos 2t \times \cos 2t \, dt \neq 0$$

Generally,
m≠n ➔ zero
m=n ➔ non-zero

- It applies to $\sin\boldsymbol{m}t \times \cos\boldsymbol{n}t$ or $\sin\boldsymbol{m}t \times \sin\boldsymbol{n}t$.
- We say two sinusoidal signals are **orthogonal** over the [0, T] range when the integral is 0.

Circuit With Two Sources

- Total response can be derived with superposition as

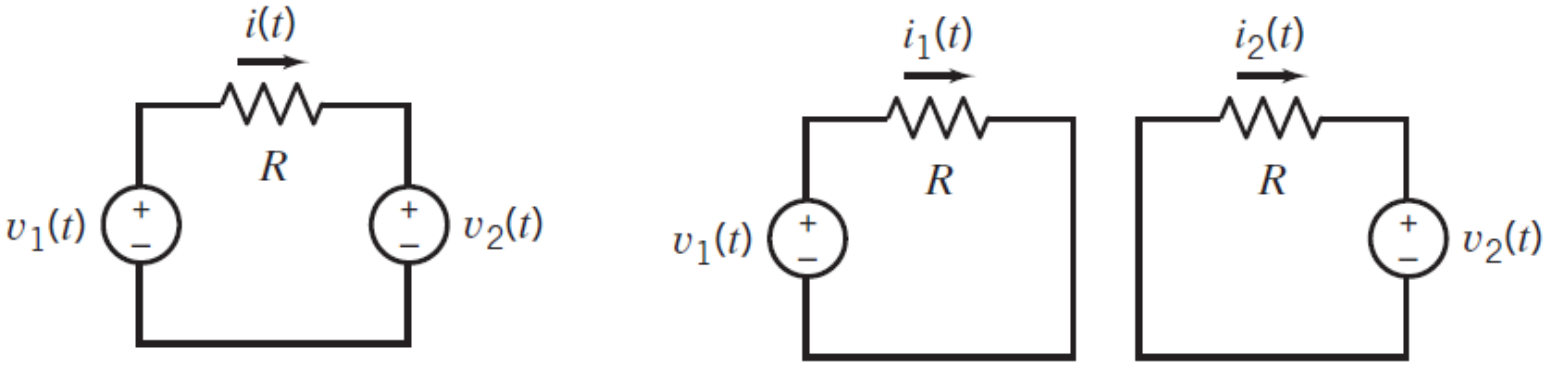
$$i = i_1 + i_2$$

- Instantaneous power is

$$p = i^2 R = R(i_1 + i_2)^2 = R\left(i_1^2 + i_2^2 + 2i_1 i_2\right)$$

- Average power is

$$\begin{aligned} P &= \frac{1}{T}\int_0^T p\,dt = \frac{R}{T}\int_0^T \left(i_1^2 + i_2^2 + 2i_1 i_2\right) dt \\ &= \frac{R}{T}\int_0^T i_1^2\,dt + \frac{R}{T}\int_0^T i_2^2\,dt + \frac{2R}{T}\int_0^T i_1 i_2\,dt = P_1 + P_2 + \frac{2R}{T}\int_0^T i_1 i_2\,dt \end{aligned}$$



When v_1 & v_2 Frequency Differs

- How can you determine T for average power?

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{R}{T}\int_0^T \left(i_1^2 + i_2^2 + 2i_1 i_2\right) dt$$
$$= \frac{R}{T}\int_0^T i_1^2\,dt + \frac{R}{T}\int_0^T i_2^2\,dt + \frac{2R}{T}\int_0^T i_1 i_2\,dt = P_1 + P_2 + \frac{2R}{T}\int_0^T i_1 i_2\,dt$$

- Then,

$$\frac{2R}{T}\int_0^T i_1 i_2\,dt = 0$$

- Which means

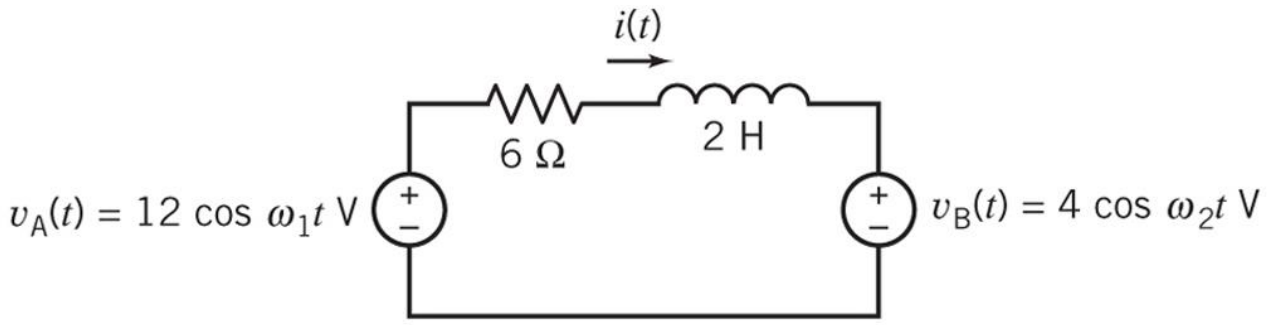
$$P = P_1 + P_2$$ ➔ **Power Superposition !**

- Note that principle of power superposition is valid only when

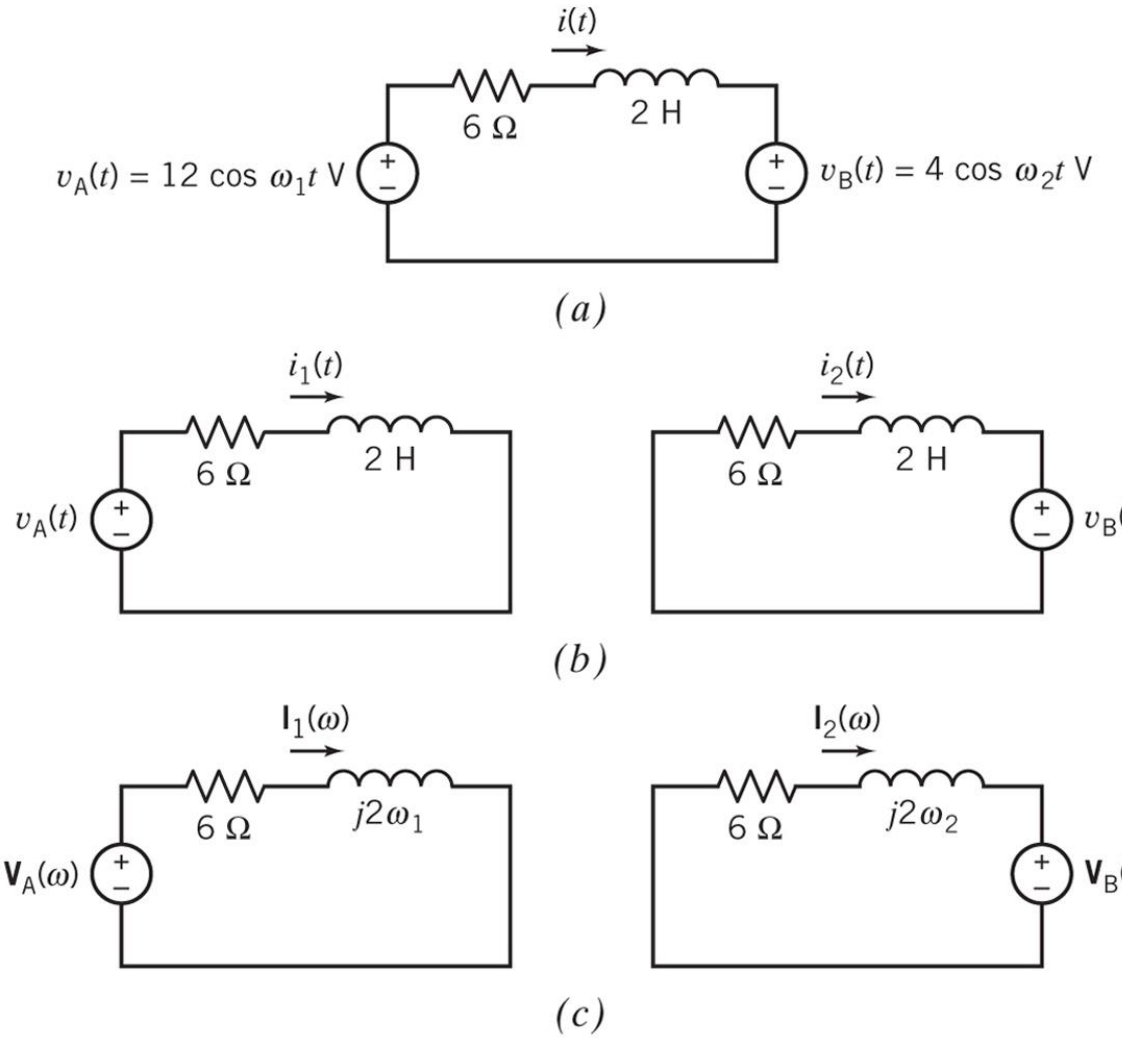
$$\frac{2R}{T}\int_0^T i_1 i_2\,dt = 0$$

Example 11.7-1

- To illustrate power superposition, consider two cases:
  1) $v_A(t) = 12 \cos 3t$ V, $v_B(t) = \cos 4t$ V
  2) $v_A(t) = 12 \cos 4t$ V, $v_B(t) = \cos 4t$ V
- Find the average power absorbed by the 6-Ω resistor.



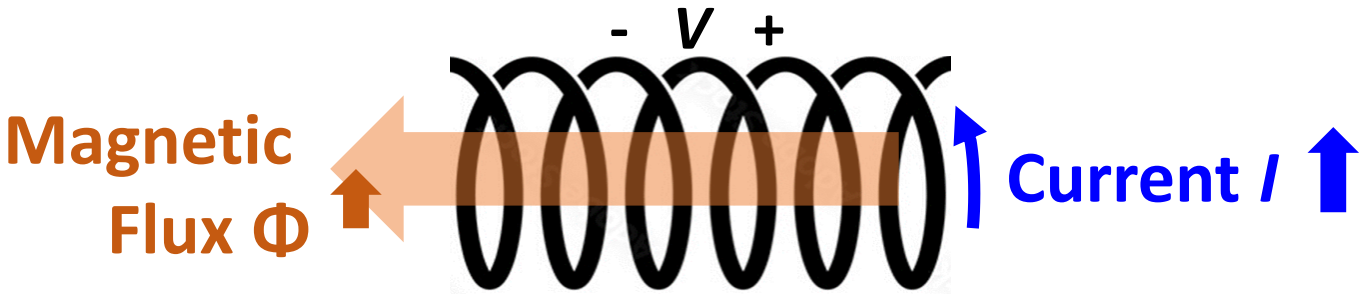
Example 11.7-1



Example 11.7-1

Review on Inductor Basic Physics

- Suppose we have an inductor (in the form of coil):



$\Phi = \square \times i = c \times N \times i$

N : number of turns
c : magnetic properties

$i \rightarrow \Phi$

$di/dt \rightarrow d\Phi/dt \rightarrow V$

$V \propto d\Phi/dt$

Faraday's law

$V = \square \times d\Phi/dt$
$= N \times d\Phi/dt$

$v(t) = L \times \frac{di(t)}{dt}$: Definition of inductance

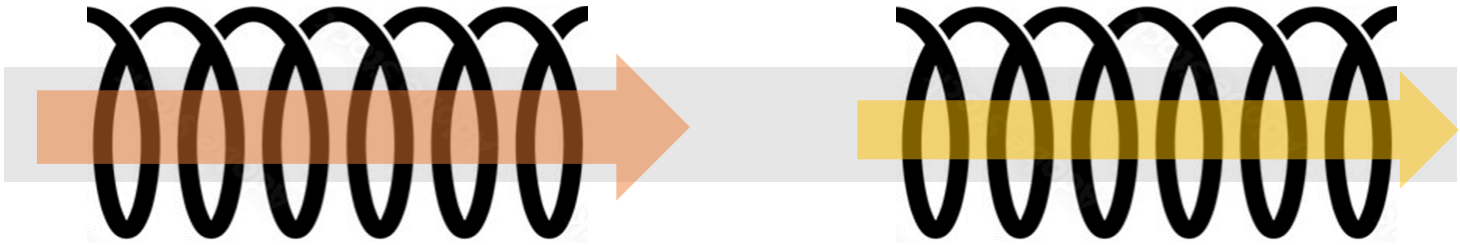
- Now we are ready to look into L (inductance) in more detail

$$v(t) = \square \times \frac{di(t)}{dt} = c \times N \times N \times \frac{di(t)}{dt} \rightarrow L = cN^2$$

Coupled Inductors & Mutual Inductance

Primary Coil | **Secondary Coil**

Coil 1 | Coil 2



di_1/dt $\times c_1 \times N_1$ $d\Phi_{11}/dt$ $\times N_1$ v_1

$\times c_{12} \times N_1$

$d\Phi_{12}/dt$ $\times N_2$ v_2

$$v_1 = \square \times \frac{di_1(t)}{dt}$$

$$\square = L_1 = c_1 N_1^2$$

$$v_2 = \square \times \frac{di_1(t)}{dt}$$

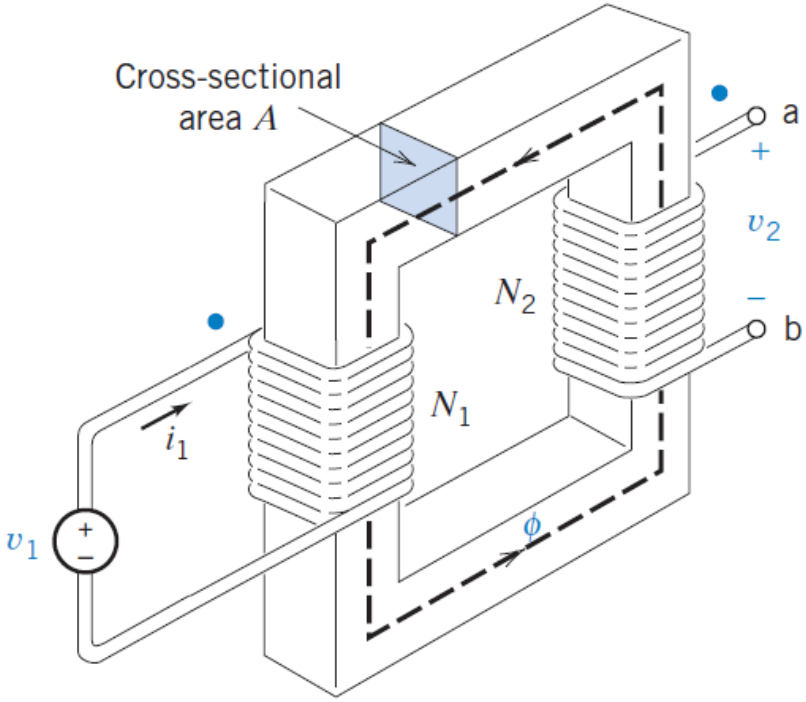
$$\square = c_{12} N_1 N_2 = M_{12}$$

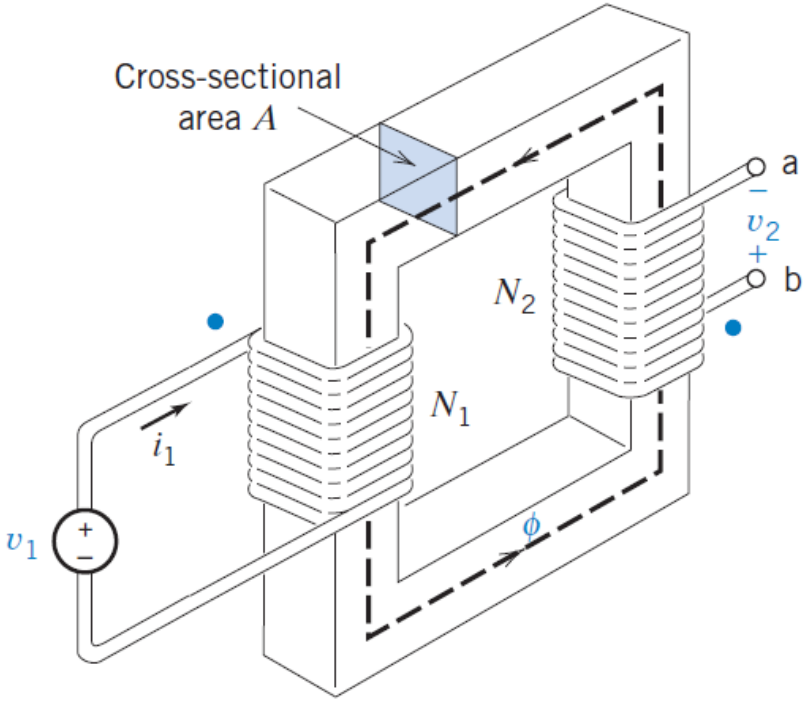
(mutual inductance)

Relates $i_1 \rightarrow v_2$

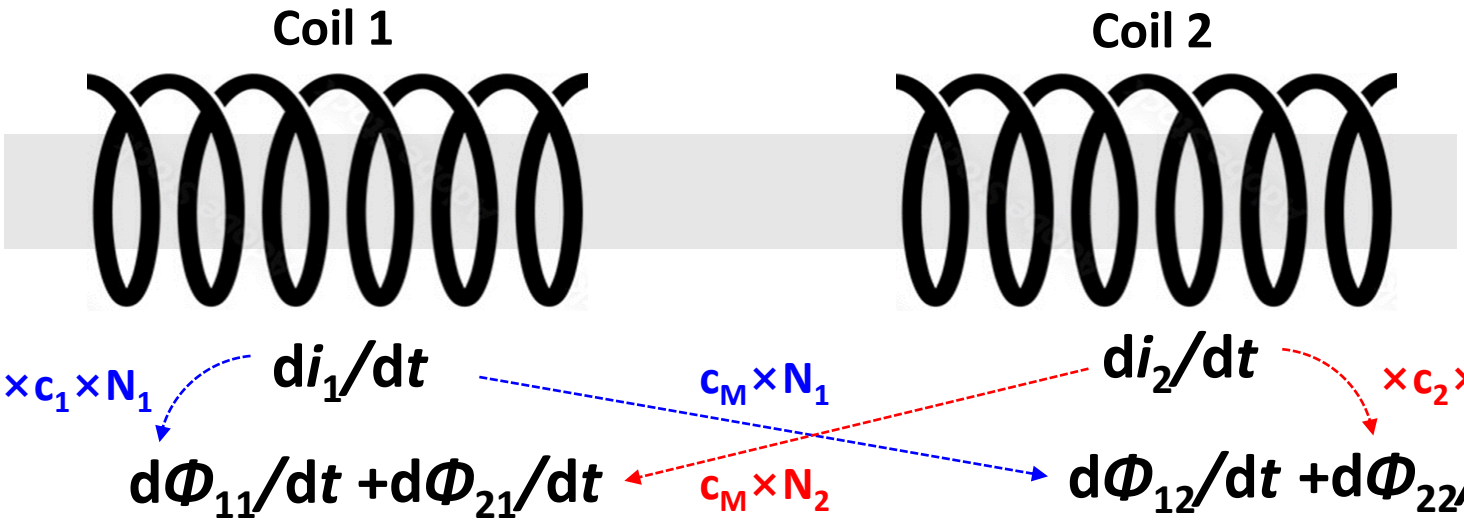
Another form of Coupled Inductors

- **Coupled inductors**, or coupled coils, are magnetic devices that consist of two or more multiturn coils wound on a common core.





More Complex Situation;
Both Coils Generates Current Change



$$v_1 = N_1 \times (d\Phi_{11}/dt + d\Phi_{21}/dt)$$
$$= L_1 di_1/dt + M di_2/dt$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

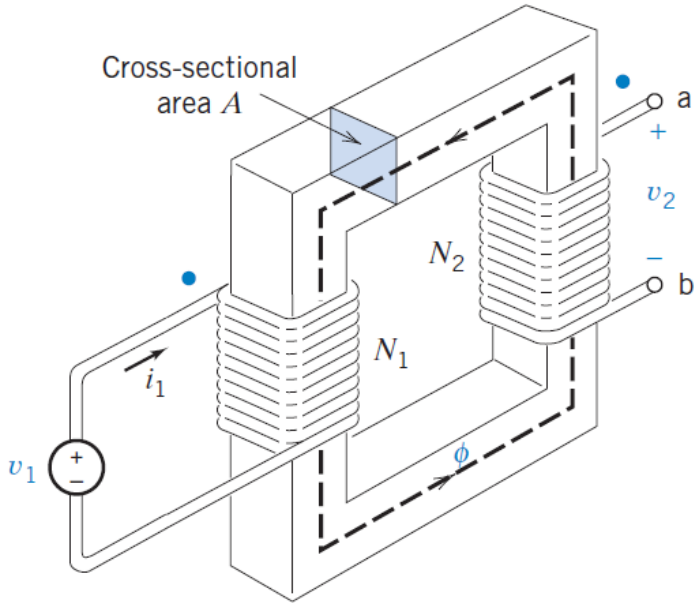
$$v_2 = N_2 \times (d\Phi_{12}/dt + d\Phi_{22}/dt)$$
$$= L_2 di_2/dt + M di_1/dt$$

$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

Recall that in the previous slide,

1) $v_{11} = c_1 N_1^2 \times di_1/dt = L_1 di_1/dt$
2) $v_{12} = c_M N_1 N_2 \times di_1/dt = M di_1/dt$

Mutual Inductance M



- Voltage $v_1(t)$ causes a current $i_1(t)$ in coil 1:

$$v_1 = L_1 \frac{di_1}{dt}$$

- Current $i_1(t)$ causes a flux in the magnetic core.

$$\phi = c_1 N_1 i_1$$

  - c_1 : constant determined by the magnetic properties and geometry
  - N_1 : the number of turns in coil 1.

- v_1 is related to the flux by

$$v_1 = N_1 \frac{d\phi}{dt} = N_1 \frac{d}{dt}(c_1 N_1 i_1) = c_1 N_1^2 \frac{di_1}{dt} \quad \Leftarrow \quad L_1 = c_1 N_1^2$$

- v_2 at the terminals of the second coil is induced by Φ,

$$v_2 = N_2 \frac{d\phi}{dt} = c_M N_1 N_2 \frac{di_1}{dt} = M \frac{di_1}{dt}$$

Mutual Inductance, positive number, Henry unit.

More Complex Situation; Wrapped in Opposite Direction



$v_1 = N_1 \times (d\Phi_{11}/dt - d\Phi_{21}/dt)$

$= L_1 di_1/dt - M di_2/dt$

$v_2 = N_2 \times (-d\Phi_{12}/dt + d\Phi_{22}/dt)$

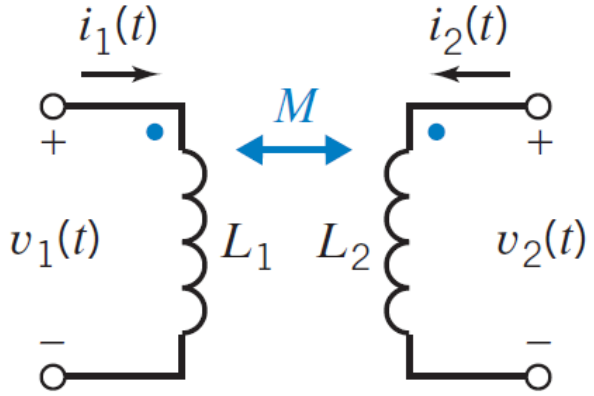
$= L_2 di_2/dt - M di_1/dt$

$V_1 = j\omega L_1 I_1 - j\omega M I_2$

$V_2 = j\omega L_2 I_2 - j\omega M I_1$

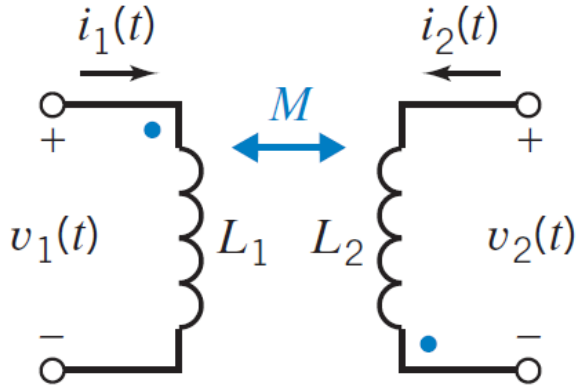
Dot Convention

- Polarity of v_2, compared to the polarity of v_1, depends on the way in which the coils are wrapped on the core : two distinct cases exist.
- **Dot convention** indicates the way the coils is wrapped on the coil.



both coil currents enter
the dotted ends of the coils

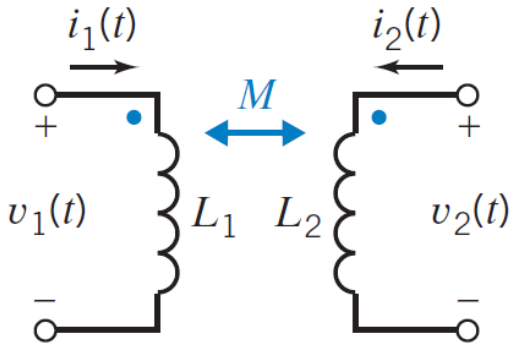
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



i1 enters the dotted end of a coil,
but i2 enters the undotted end

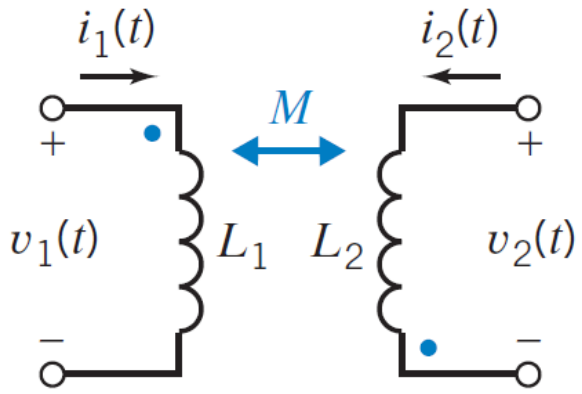
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Phasor Equations for Mutual Inductance



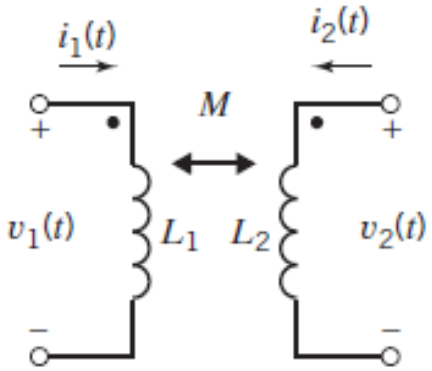
$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M\, \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M\, \mathbf{I}_1$$



$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 - j\omega M\, \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 - j\omega M\, \mathbf{I}_1$$



Coupling Coefficient

- L_1, L_2 and M each depend on the magnetic properties and geometry of the core and the number of turns in the coils.
- We can write

$$L_1 L_2 = \left(c_1 N_1^2\right)\left(c_2 N_2^2\right) = c_1 c_2 (N_1 N_2)^2 = \left(\frac{c_M N_1 N_2}{k}\right)^2 = \frac{M^2}{k^2}$$

  where $k = c_M / \sqrt{c_1 c_2}$ is coupling coefficient.

- Or

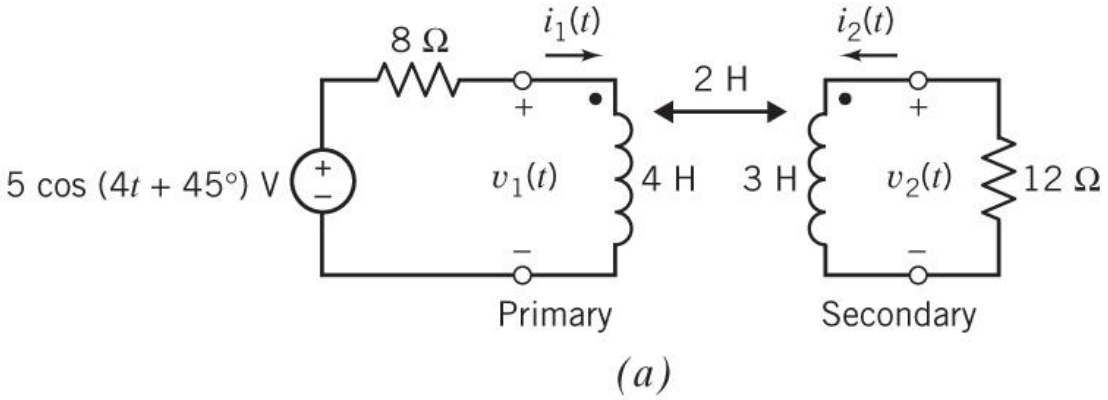
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

where

$$0 \leq k \leq 1$$

Example 11.9-1

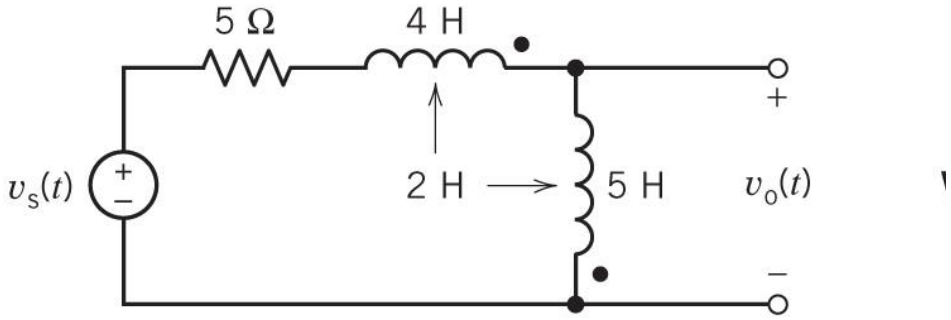
- Find $v_2(t)$



(a)

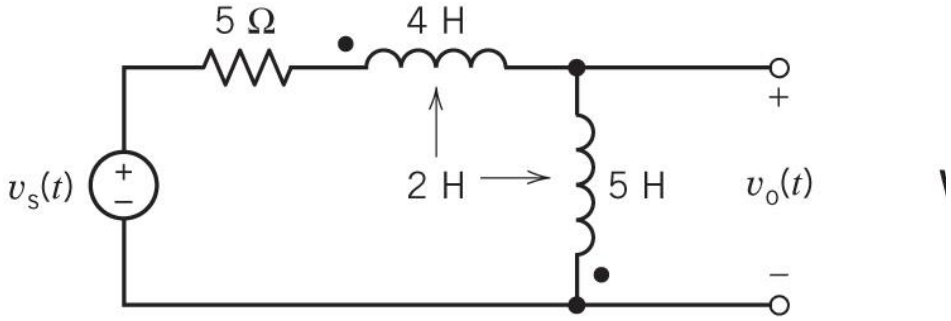
Example 11.9-2

- $v_s(t) = 5.94 \cos(3t+140°)$. Find $v_o(t)$.

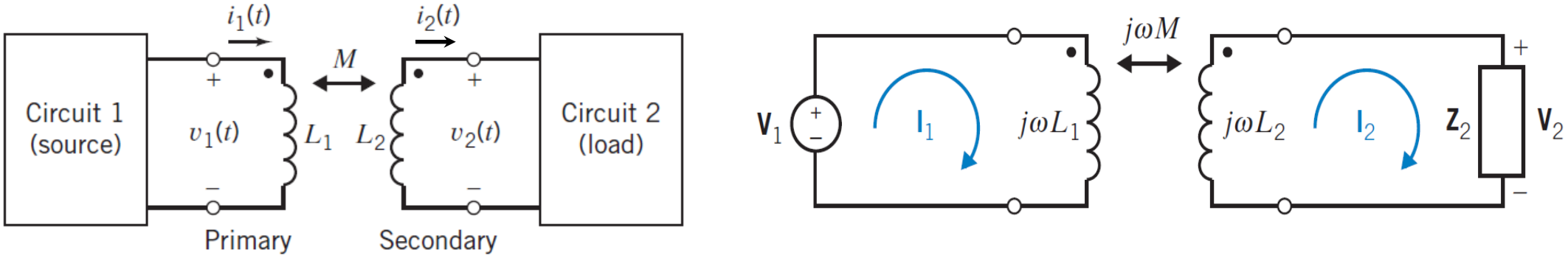


Example 11.9-3

- $v_s(t) = 5.94 \cos(3t+140°)$. Find $v_o(t)$.



Coupled Inductor as a Transformer



- Circuit analysis using node or mesh equations:

Assuming k=1

$$j\omega L_1\mathbf{I}_1 - j\omega M\,\mathbf{I}_2 = \mathbf{V}_1$$
$$-j\omega M\,\mathbf{I}_1 + (j\omega L_2 + \mathbf{Z}_2)\mathbf{I}_2 = 0$$

$$\Rightarrow \quad \mathbf{I}_2 = \left[\frac{j\omega M}{j\omega L_1\mathbf{Z}_2}\right]\mathbf{V}_1 = \left[\frac{j\omega\sqrt{L_1L_2}}{j\omega L_1\mathbf{Z}_2}\right]\mathbf{V}_1 = \frac{\sqrt{L_2}}{\mathbf{Z}_2\sqrt{L_1}}\mathbf{V}_1$$

- V_2 across Z_2, which relates V_1 to V_2 as

$$\mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2 = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 \quad \Leftarrow \quad \frac{L_2}{L_1} = \frac{c_2N_2^2}{c_1N_1^2}$$

Same core →

$$\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = n^2$$

$n = N_2/N_1$

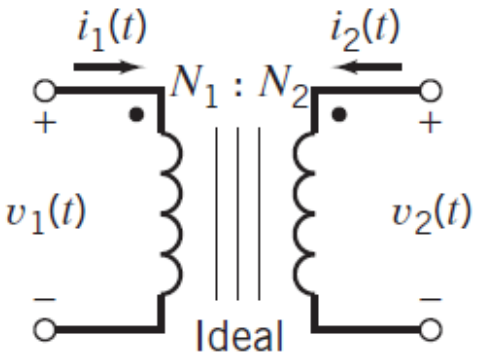
- Thus,

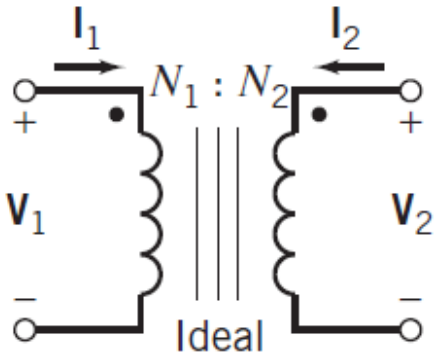
$$\mathbf{V}_2 = n\mathbf{V}_1$$

Ideal Transformer

- An ideal transformer is a model of a transformer with a coupling coefficient (k) equal to unity.
- **No power loss in the transformer**

➔ zero complex power, zero average power, and zero reactive power are absorbed by the ideal transformer



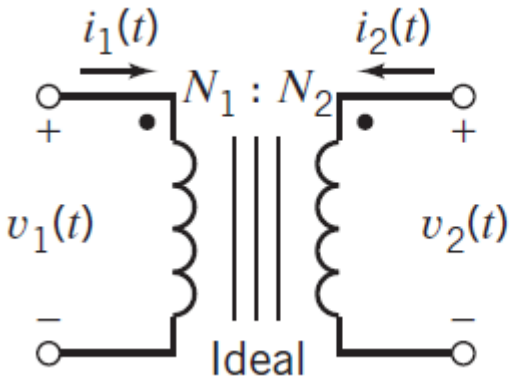


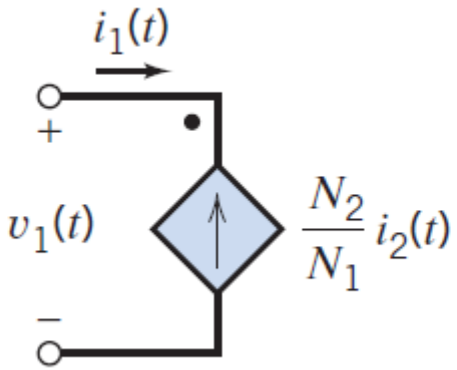
$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

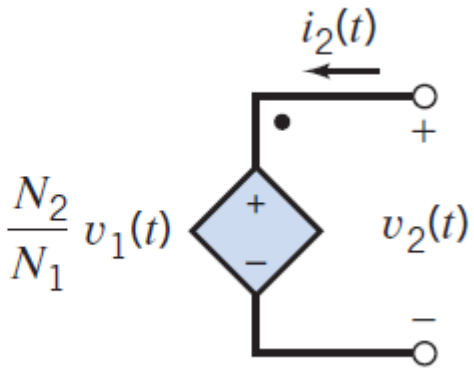
$$\mathbf{V}_2 = \frac{N_2}{N_1} \mathbf{V}_1$$

$$i_2(t) = -\frac{N_1}{N_2} i_1(t)$$

$$\mathbf{I}_2 = -\frac{N_1}{N_2} \mathbf{I}_1$$

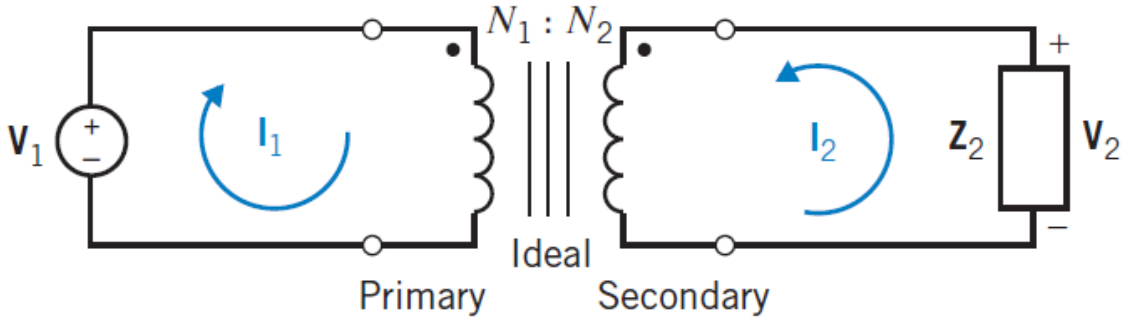






Z_1 vs. Z_2

- Consider the following circuit:



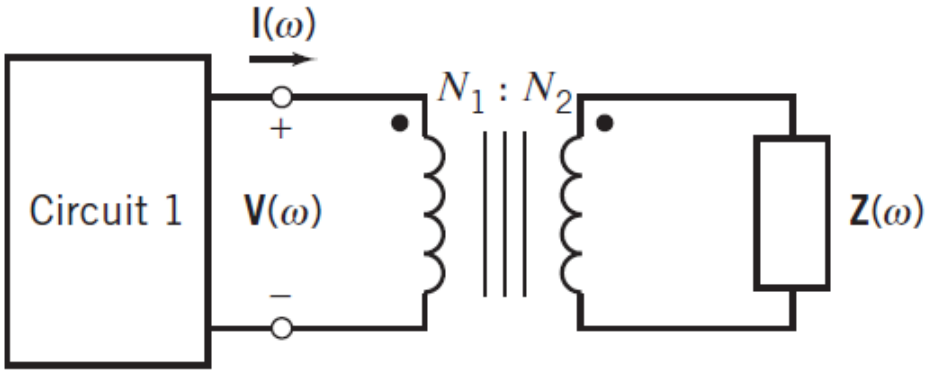
- Z_1 is called impedance seen at the primary of the transformer, or the impedance seen by the voltage source.

$$\mathbf{Z}_1 = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$ ➔ How it would be related with Z_2?

- We can write as:

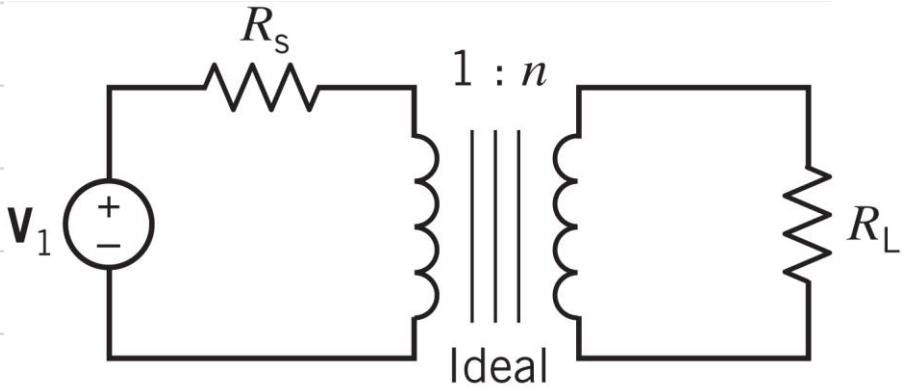
$$\mathbf{Z}_1 = \frac{\frac{N_1}{N_2}\mathbf{V}_2}{-\frac{N_2}{N_1}\mathbf{I}_2} = -\left(\frac{N_1}{N_2}\right)^2 \frac{\mathbf{V}_2}{\mathbf{I}_2} \quad \Rightarrow \quad \mathbf{Z}_1 = \left(\frac{N_1}{N_2}\right)^2 \mathbf{Z}_2$$

Equivalent Circuit For Ideal Transformer



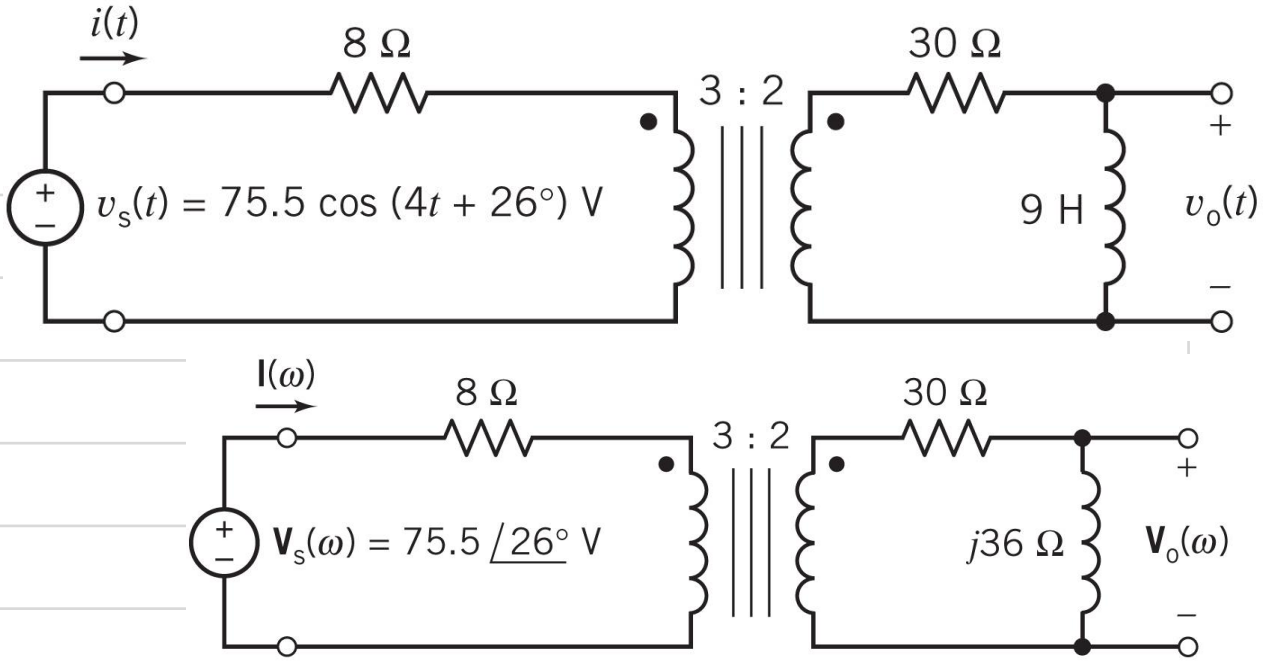
Example 11.10-1

- Find the turns ration n that is required to cause maximum power to be transferred to the load when R_L = 8 Ω and R_S = 48 Ω.

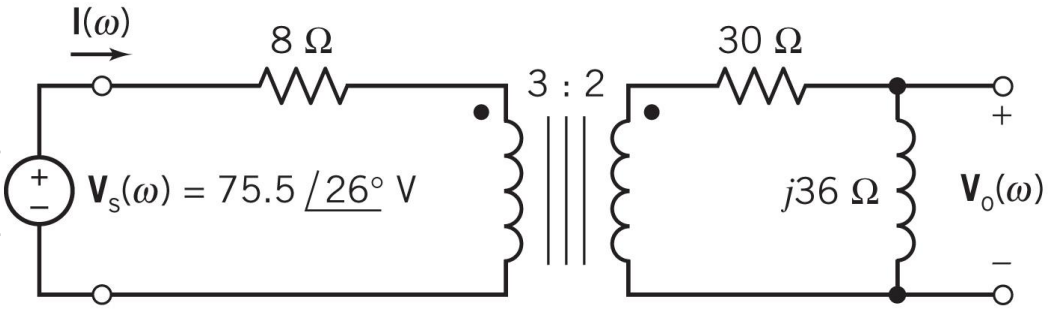


Example 11.10-2

- Determine $v_o(t)$.

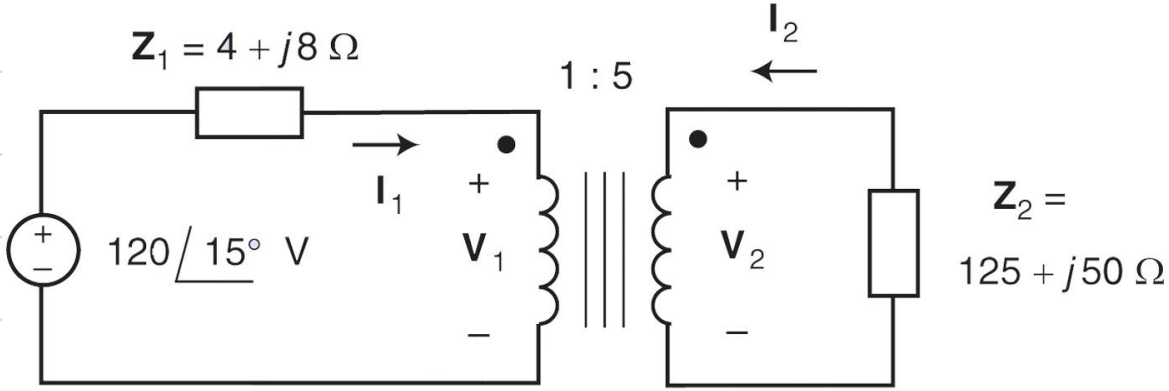


Example 11.10-2



Example 11.10-3

- Determine the value of the power (a) supplied by the voltage source, (b) received by the impedance Z_2, and (c) received by the transformer



Example 11.10-3

