Chap. 10

Sinusoidal Steady-State Analysis (2)

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Revisit What We Learned

- In R only circuit, we can derive any I & V through

  1) Ohm's law **2) KVL & KCL**

- In RLC circuit,

  1) Ohm's law : R **2) KVL & KCL**

  Differential Eq. : LC

- In AC circuit, by fixing "**cos(ωt+**" part, the differential equation can be replaced w/ the simple equations.

  1) Ohm's law : R +LC **2) KVL & KCL**

  Phasor form: $\mathbf{V} = \square \times \mathbf{I}$ ➔ $\mathbf{V} = A_V exp(j\theta_V)$, $\mathbf{I} = A_I exp(j\theta_I)$

  ↑

  R, 1/jωC, jωL

Time-Domain vs. Frequency Domain ,and Impedance

Original world

$$v(t) = A_V \cos(\omega t + \theta_V)$$

$$i(t) = A_I \cos(\omega t + \theta_I)$$

Our world

$$V(\omega) = A_v \, exp(j\theta_V)$$

$$I(\omega) = A_I exp(j\theta_I)$$

Don't forget!
We can use KVL & KCL

R

C

L

$Z_R(\omega) = R$

$Z_C(\omega) = 1/j\omega C$

$Z_L(\omega) = j\omega L$

➔ **Impedance *Z***

Ohm's law in AC circuit:
$V(\omega) = Z(\omega) \times I(\omega)$

Original form **vs.** **Phasor form**

Time-domain **vs.** **Frequency-domain**

We omit "cos(ωt+"

Understanding Impedance

- ✓ 10.4 Impedances
- ✓ 10.5 Series and Parallel Impedances

Motivation

- Think of a black box as follows.



What if v_{AB} = **12 cos(200t + 50) V** and i_{AB} = **3 cos(200t + 50) mA**?

- What can we say for the black box?

“Resistance”

- With the resistance value, we can simply derive the voltage from the current or vice versa.
- In the phasor representation of the current, we can simply do the same thing even for the **capacitance** and the **inductance** by utilizing

“Impedance”

Ohm's Law for AC circuit & Impedance

- Consider that

$$v(t) = V_{\mathrm{m}} \cos(\omega t + \theta) \ \mathrm{V} \ \text{and} \ \ i(t) = I_{\mathrm{m}} \cos(\omega t + \phi) \ \mathrm{A}$$

- The above can be represented as

$$\mathbf{V}(\omega) = V_{\mathrm{m}} \angle \theta \ \text{and} \ \mathbf{I}(\omega) = I_{\mathrm{m}} \angle \phi$$

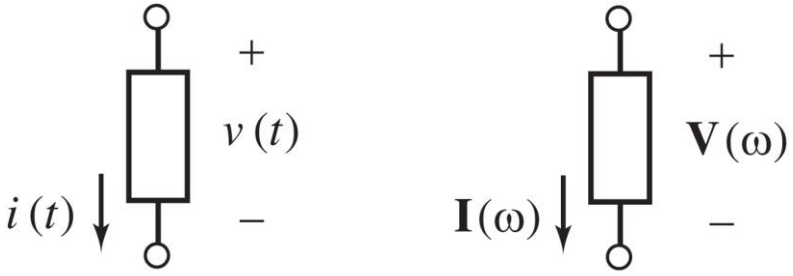
- The **impedance** of an element of an ac circuit is defined to be the ratio of the voltage phasor to the current phasor.

$$\mathbf{Z}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)} = \frac{V_{\mathrm{m}} \angle \theta}{I_{\mathrm{m}} \angle \phi} = \frac{V_{\mathrm{m}}}{I_{\mathrm{m}}} \angle (\theta - \phi) \ \Omega$$

- Consequently,

$$\mathbf{V}(\omega) = \mathbf{Z}(\omega) \, \mathbf{I}(\omega)$$

which is Ohm's law for ac circuits.



Impedance of a Capacitor

- Consider the voltage applied to a cap,

$$v_C(t) = A\cos(\omega t + \theta)$$

- The corresponding cap current is

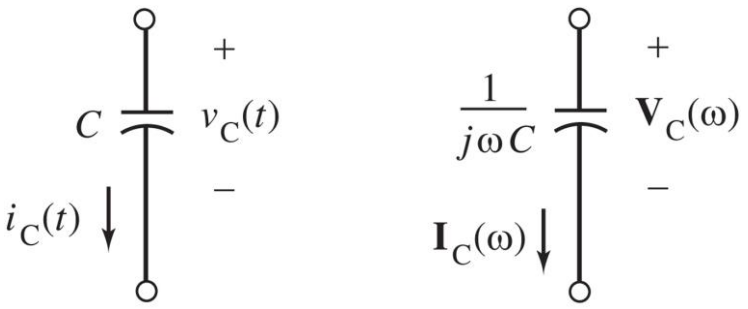
$$i_C(t) = -\omega C \times A\sin(\omega t + \theta) = \omega C \times A\cos(\omega t + \theta + 90^o)$$

- The phasor voltage and current is

$$\mathbf{V_C}(\omega) = A\angle\theta$$
$$\mathbf{I_C}(\omega) = \omega C \times A\angle(\theta + 90^o) = \omega C\angle 90^o \times A\angle\theta = j\omega C \times A\angle\theta$$

- See Ohm's law in AC circuit, $\mathbf{V}(\omega) = \mathbf{Z}(\omega)\,\mathbf{I}(\omega)$
- The impedance of a capacitance (frequency dependency?)

$$\mathbf{Z_C}(\omega) =$$



Impedance of an Inductor

- Consider the voltage applied to a cap,

$$i_L(t) = A\cos(\omega t + \theta)$$

- The corresponding cap current is

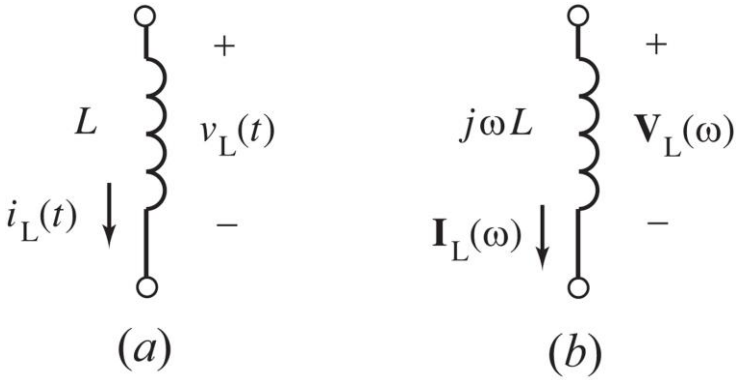
$$v_L(t) = -\omega L \times A\sin(\omega t + \theta) = \omega L \times A\cos(\omega t + \theta + 90^o)$$

- The phasor voltage and current is

$$\mathbf{I_L}(\omega) = A\angle\theta$$
$$\mathbf{V_L}(\omega) = \omega L \times A\angle(\theta + 90^o) = \omega L\angle 90^o \times A\angle\theta = j\omega L \times A\angle\theta$$

- The impedance of a inductance (frequency dependency?)

$$\mathbf{Z_L}(\omega) =$$



Impedance of a Resistor

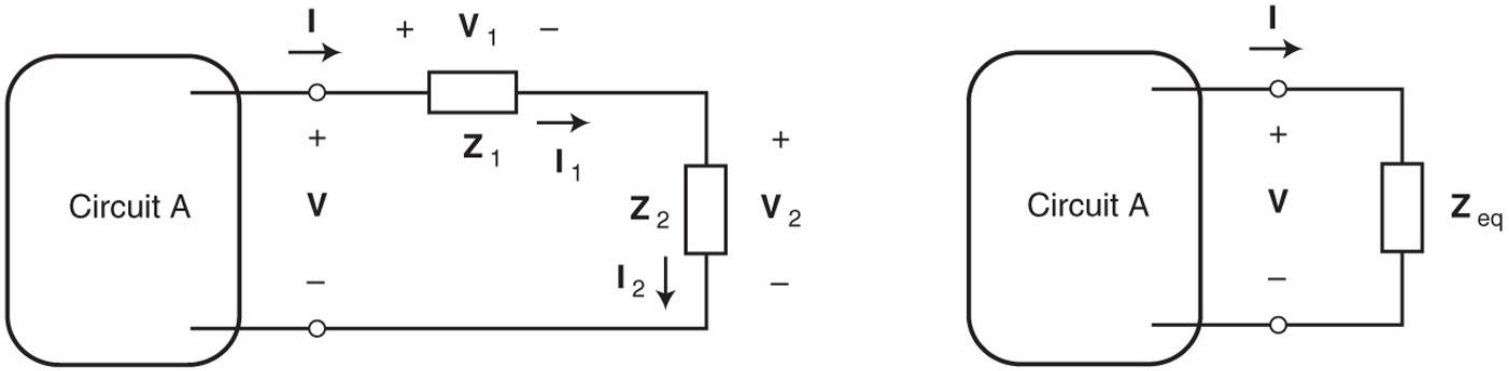
- One can easily derive that

$$\mathbf{Z}_{\mathrm{R}}(\omega) = \frac{\mathbf{V}_{\mathrm{R}}(\omega)}{\mathbf{I}_{\mathrm{R}}(\omega)} = \frac{A\angle\theta}{\frac{A}{R}\angle\theta} = R \ \ \Omega$$

$$\mathbf{V}_{\mathrm{R}}(\omega) = R\,\mathbf{I}_{\mathrm{R}}(\omega)$$

Series Impedances

- Series Impedances



$$\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = (\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}$$

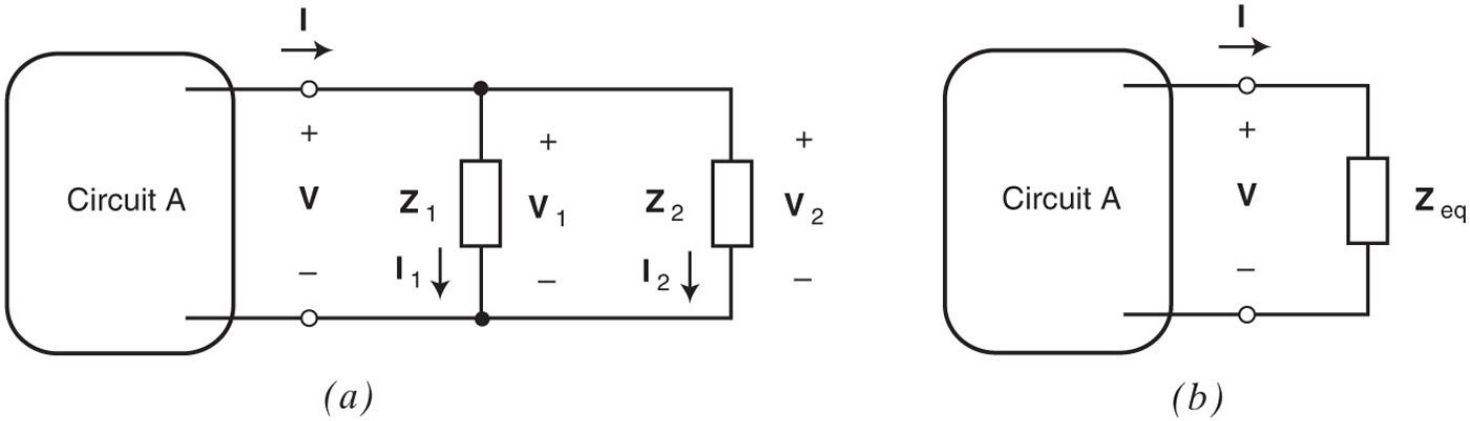
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2$$

- In general,

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_n$$

Parallel Impedances

- Parallel Impedances

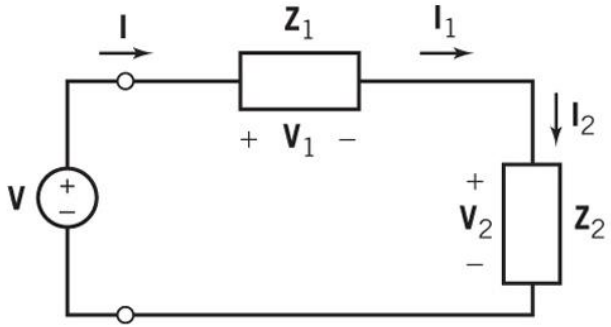
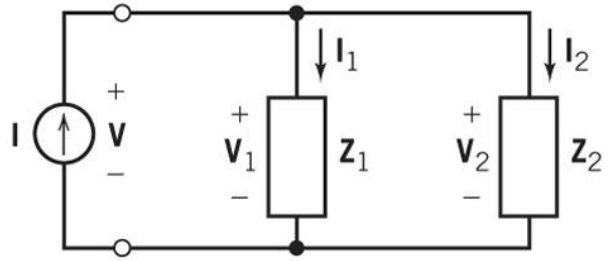


$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right)\mathbf{V}$$

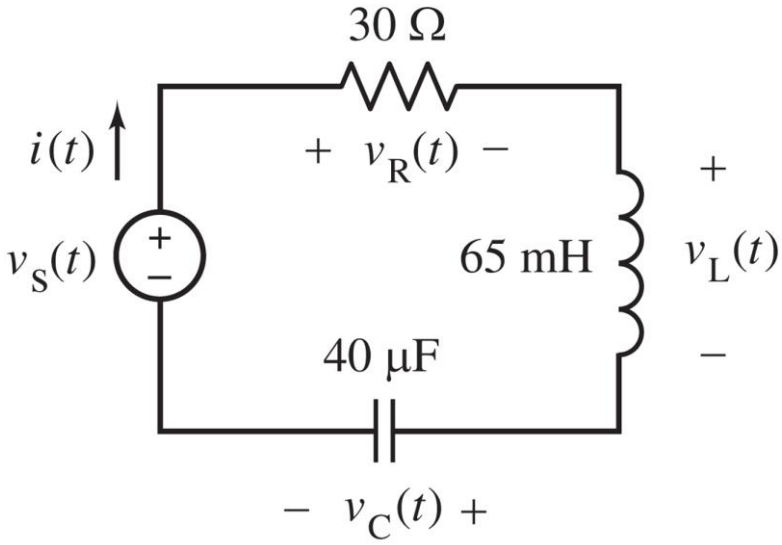
- In general,

$$\mathbf{Z}_{\text{eq}} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_n}}$$

Voltage & Current Division in Frequency Domain

	CIRCUIT	EQUATIONS
Voltage division		$\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}$ $\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$ $\mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$
Current division		$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V}$ $\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$ $\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$

Example 10.4-1



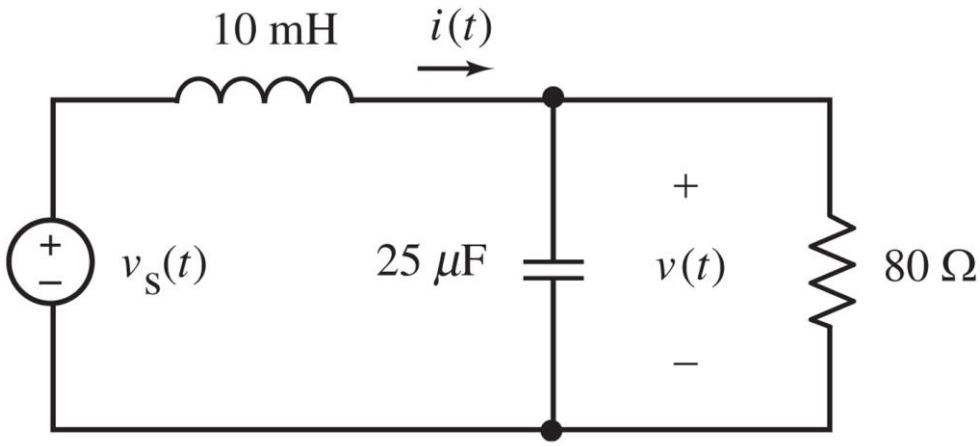
- Source voltage

$$v_s = 12 \cos(1000t + 15^o)\ V$$

- Find the current *i*(t)

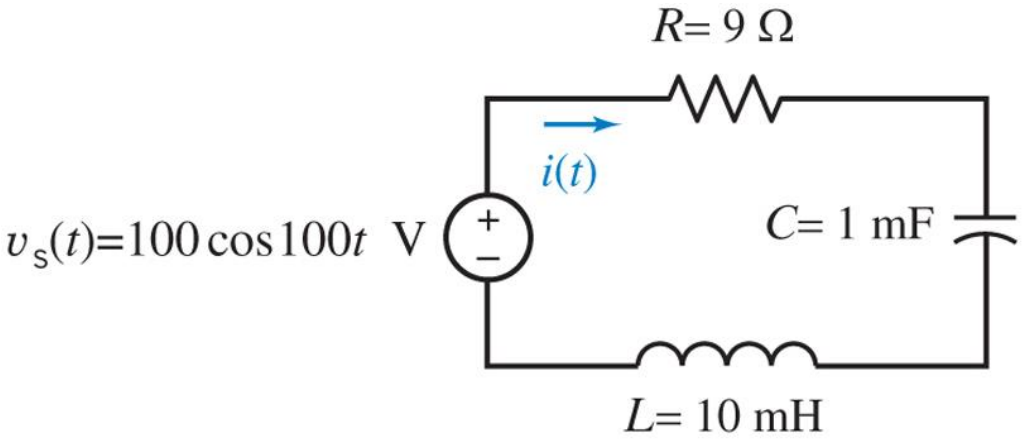
Example 10.4-2

- $v_s(t) = 48 \cos (500t + 75^o)$
- Determine $v(t)$



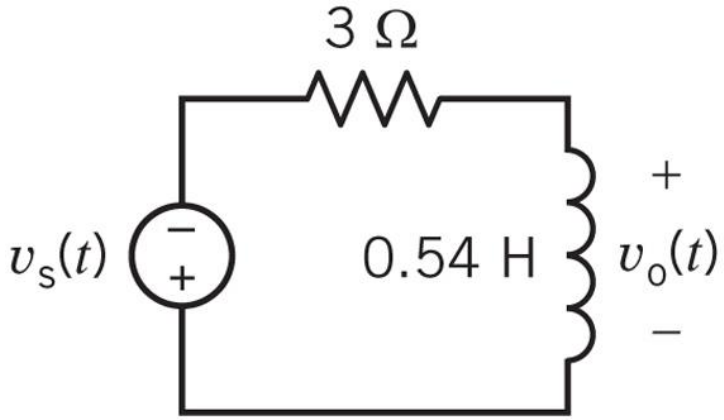
Example 10.5-1

- Determine i(t)



Example 10.5-2

- $v_s(t) = 7.28 \cos(4t + 77^o)$ V
- Find $v_o(t)$



Checkpoint

✓Definition of Ohm's law for AC circuit and impedance

✓Applying KVL, KCL to find Z_{eq}

➔ So we can analyze circuit as R-only circuit