Chap. 13

Frequency Response (2)

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Frequency Response in Circuits

- ✓ Bode plot for 2nd order LPF
- ✓ 13.4 Resonant Circuits
- ✓ 13.5 Frequency Response of Op Amp Circuits

Bode Plot

- Please remember that
  - Zeros cause the slope to increase by 20 dB/decade.
  - Poles cause the slope to decrease by 20 dB/decade.
- The magnitude drop of $1/\sqrt{2}$ results in 3dB drop.

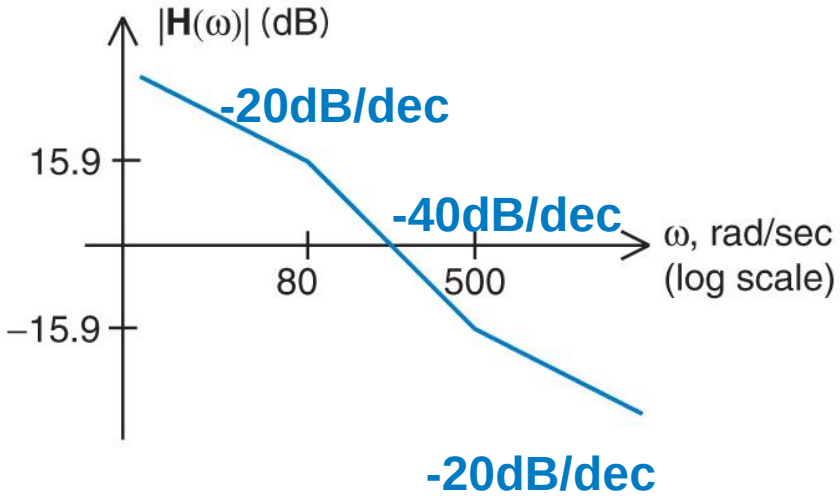
Example 13.3-1

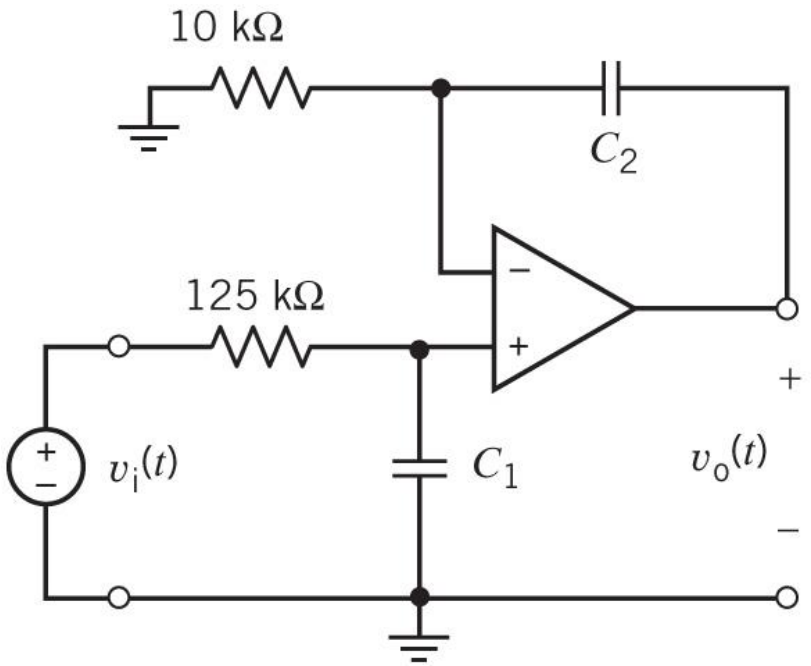
- Find the asymptotic magnitude Bode plot of

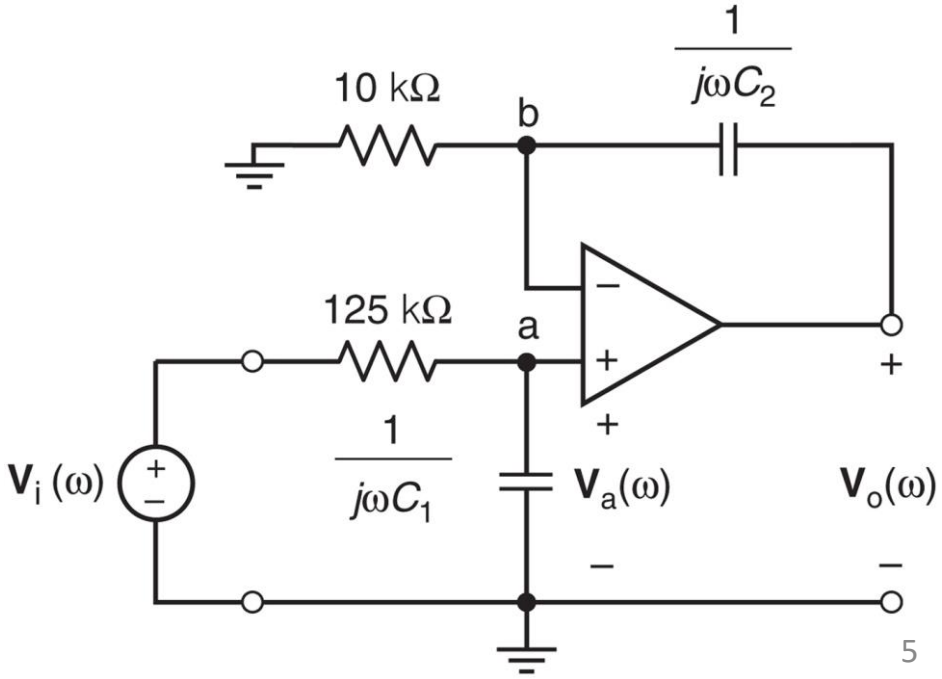
$$\mathbf{H}(\omega) = K \frac{j\omega}{1 + j\frac{\omega}{p}}$$

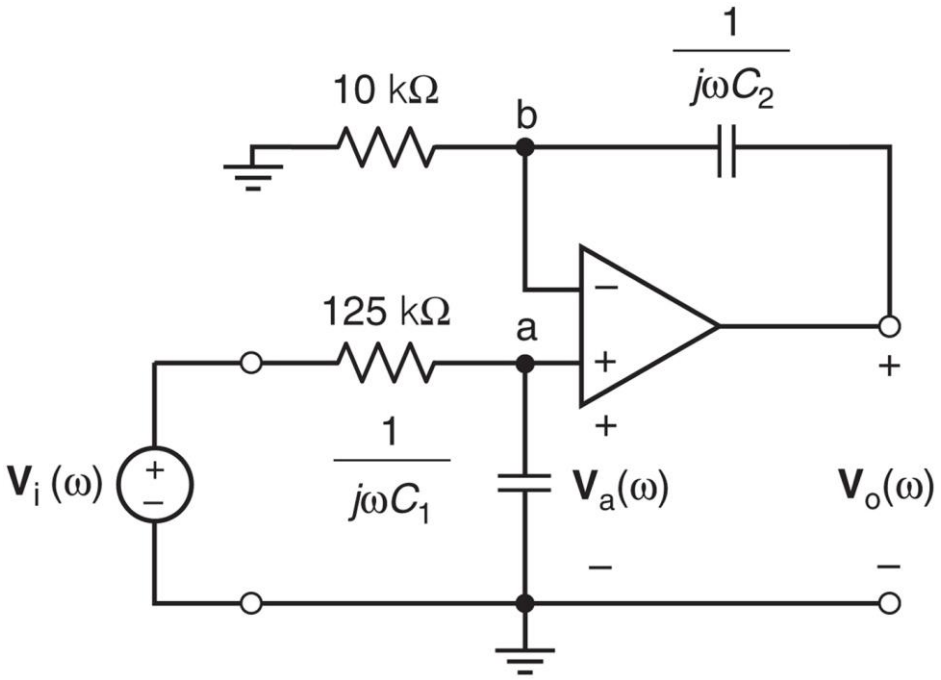
Example 13.3-2

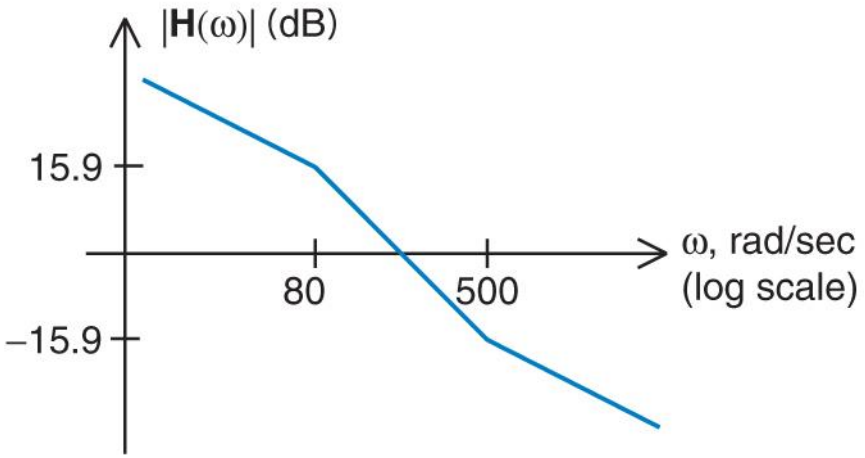
- Determine the values of C_1 and C_2.





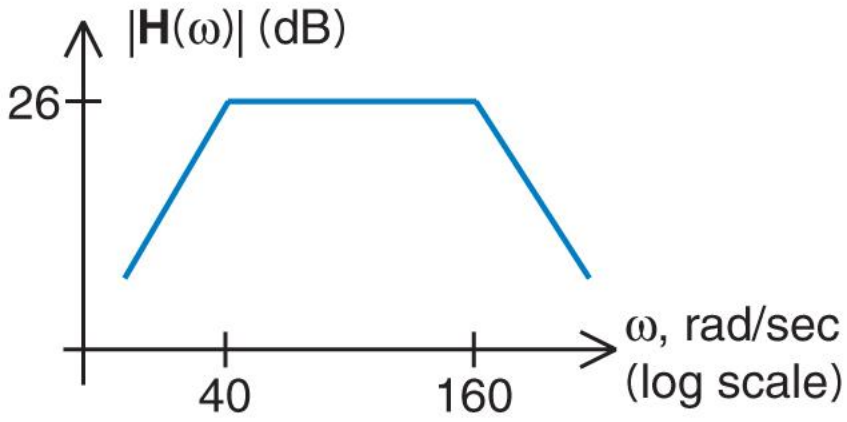


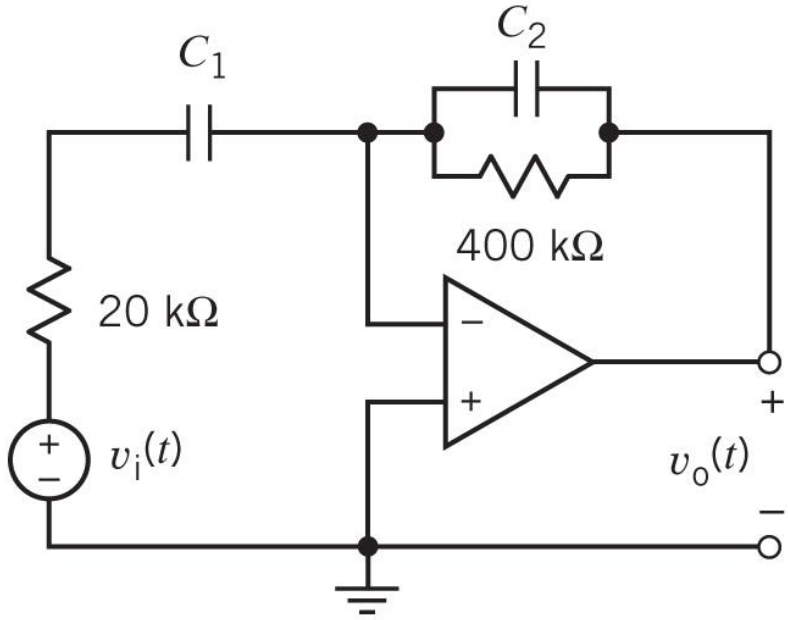

1
jωC2
10 kΩ
b
125 kΩ
a
1
jωC1
Vi (ω)
Va(ω)
Vo(ω)

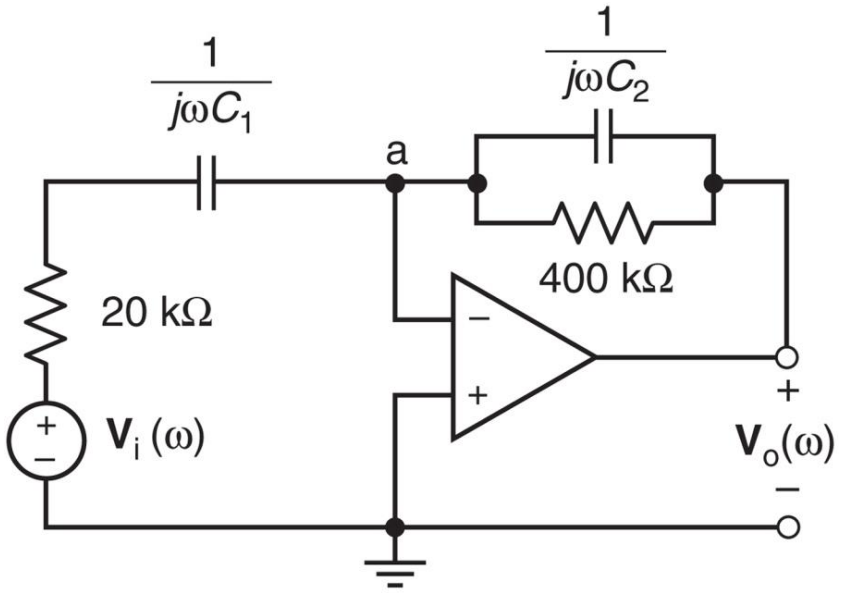

|H(ω)| (dB)
15.9
−15.9
80
500
ω, rad/sec
(log scale)

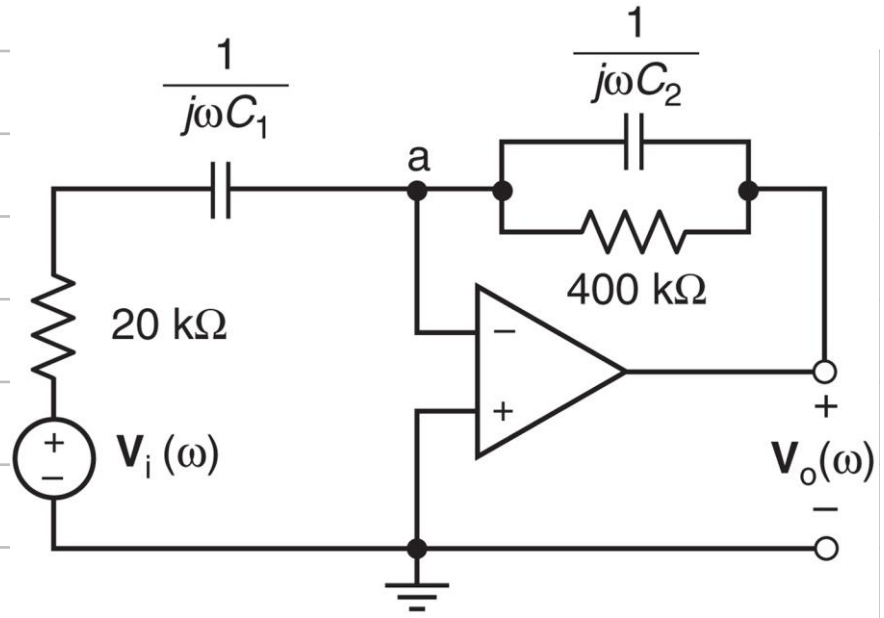
Example 13.3-3

- Determine C_1 and C_2.








1
jωC1
1
jωC2
a
400 kΩ
20 kΩ
Vi (ω)
Vo(ω)

Low Pass Filter?

- Consider the following H(ω):

$$H(\omega) = \frac{1}{\left(1+j\frac{\omega}{40}\right)\left(1+j\frac{\omega}{100}\right)} = \frac{4000}{(40+j\omega)(100+j\omega)} = \frac{4000}{(j\omega)^2 + 140j\omega + 4000}$$

- Can you draw asymptotic magnitude Bode plot?

- Can you cut more **sharply** with corner freq $\omega_0 = 40$? Make denominator as perfect square expression. Then, the slope becomes -40dB/dec :

$$H(\omega) = \frac{1}{\left(1+j\frac{\omega}{40}\right)\left(1+j\frac{\omega}{40}\right)} = \frac{1600}{(40+j\omega)(40+j\omega)} = \frac{1600}{(j\omega)^2 + 80j\omega + 1600}$$

➔ What if we cannot factorize the denominator clearly as shown above? Like when the discriminant D < 0.

Recall for the Approximation for 1st Order Denominator

- For the factorizable denominator,

$$\left\{\left(\frac{j\omega}{40}\right)^2 + 2\frac{j\omega}{40} + 1\right\} \rightarrow \left(1 + j\frac{\omega}{40}\right)\left(1 + j\frac{\omega}{40}\right)$$

- Or by multiplying 1600 as,

$$(j\omega)^2 + 80j\omega + 1600 \rightarrow (j\omega + 40)\,(j\omega + 40)$$

- We applied the approximation for the magnitude as

$$\left|1 + j\frac{\omega}{40}\right| = \sqrt{1 + \frac{\omega^2}{40^2}} \cong \begin{cases} 1 & \omega < 40 \\ \frac{\omega}{40} & \omega > 40 \end{cases}$$

$$|j\omega + 40| = \sqrt{\omega^2 + 1600} \cong \begin{cases} 40 & \omega < 40 \\ \omega & \omega > 40 \end{cases}$$

- Can we do the same when the denominator is not factorizable?

Applying for D < 0

- When D < 0,

$$(j\omega)^2 + 60j\omega + 1600 \rightarrow \left(j\omega + 30 - j\sqrt{700}\right)\left(j\omega + 30 + j\sqrt{700}\right)$$

- Using the same logic as before,

$$\left|j\omega + 30 - j\sqrt{700}\right| = \sqrt{\left(\omega - \sqrt{700}\right)^2 + 900} \cong \begin{cases} 30 & \omega - \sqrt{700} < 30 \\ \omega - \sqrt{700} & \omega - \sqrt{700} > 30 \end{cases}$$

$$\left|j\omega + 30 + j\sqrt{700}\right| = \sqrt{\left(\omega + \sqrt{700}\right)^2 + 900} \cong \begin{cases} 30 & \omega + \sqrt{700} < 30 \\ \omega + \sqrt{700} & \omega + \sqrt{700} > 30 \end{cases}$$

- It is still complex to draw plot in dB → There is no reason for using this approximation.

Approximation for D < 0

- We can directly apply the approximation for 2nd order expression **without factorizing** it as follows.
- First, obtaining magnitude as

$$|(j\omega)^2 + 60j\omega + 1600| = |1600 - \omega^2 + 60j\omega| = \sqrt{(1600 - \omega^2)^2 + 60\omega^2}$$

- Then, we apply more rough approximation as

$$\sqrt{(1600 - \omega^2)^2 + (60\omega)^2} \cong \begin{cases} 1600 & \omega < 40 \\ \omega^2 & \omega > 40 \end{cases}$$

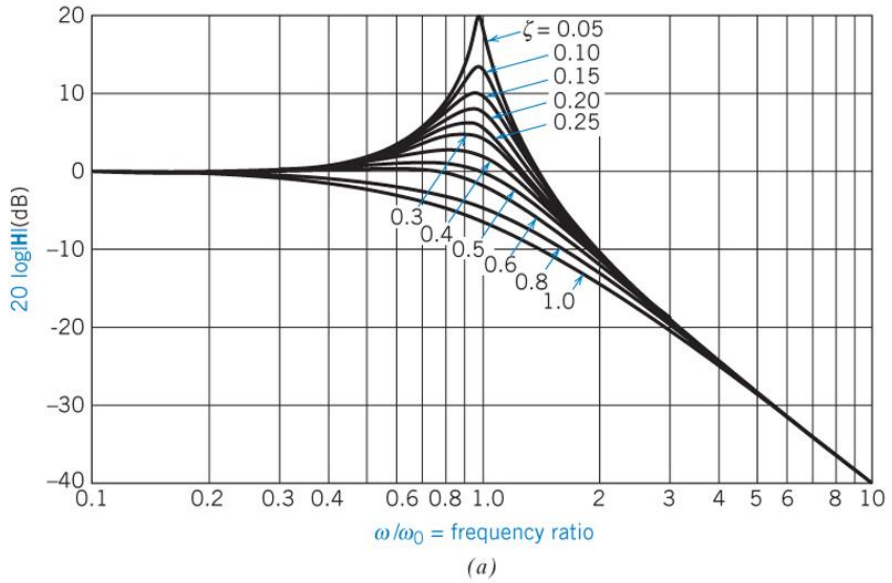
- We can use it for D < 0 to draw Bode Plot.

Example 13.3-4

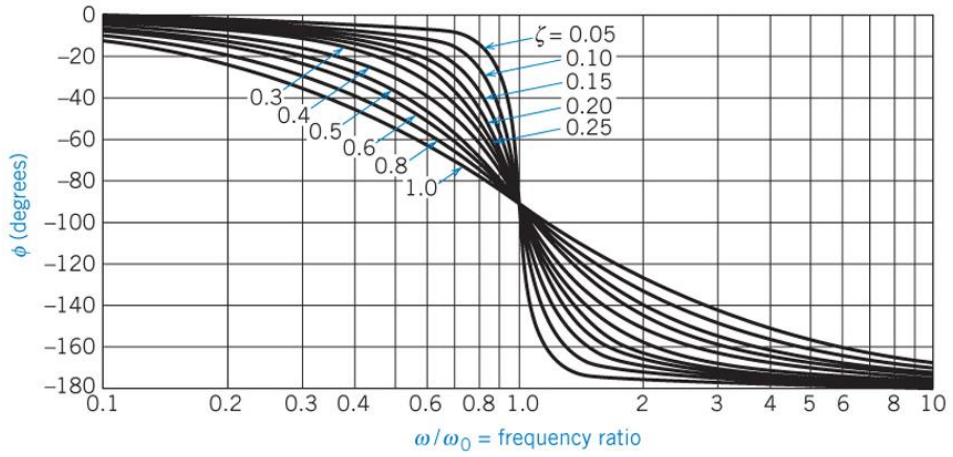
- Draw the asymptotic magnitude Bode plot for the following network function when $\zeta < 1$ and dc gain = 1.

$$\mathbf{H}(\omega) = \frac{k\,\omega_0^2}{(j\omega)^2 + j2\zeta\omega_0\omega + \omega_0^2}$$

Example 13.3-4



(a)



(b)

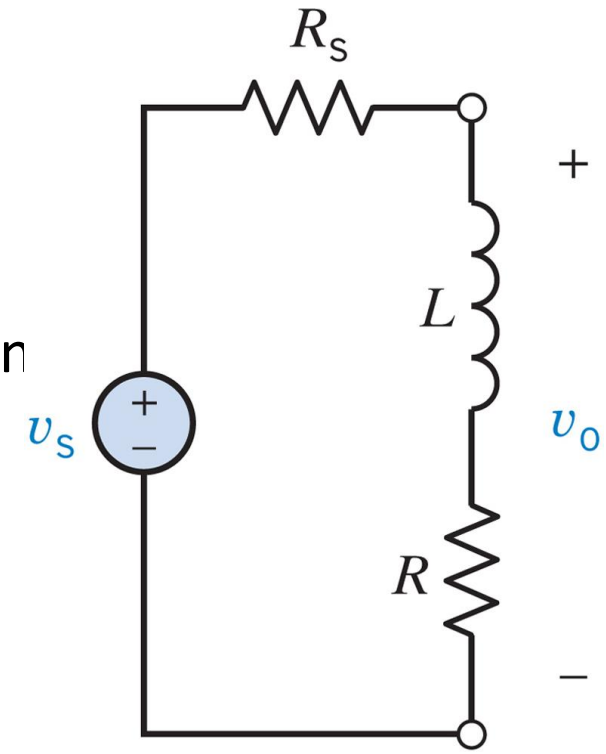
Example 13.3-5

- Find the asymptotic magnitude Bode plot of

$$\mathbf{H}(\omega) = \frac{5(1 + 0.1j\omega)}{j\omega(1 + 0.5j\omega)\left[1 + 0.6\left(\frac{j\omega}{50}\right) - \left(\frac{\omega}{50}\right)^2\right]}$$

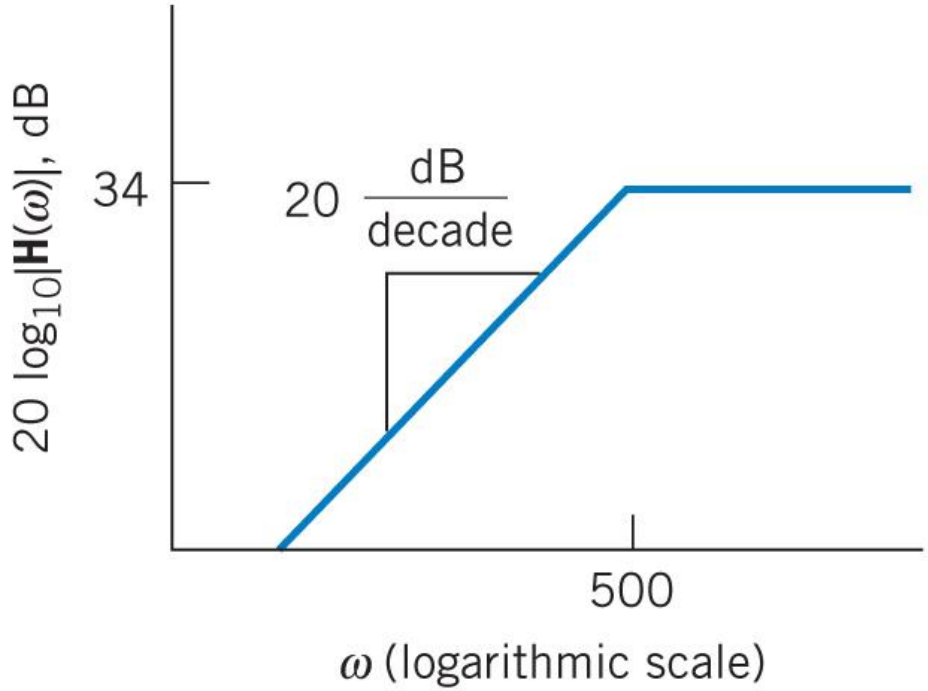
Example 13.3-6

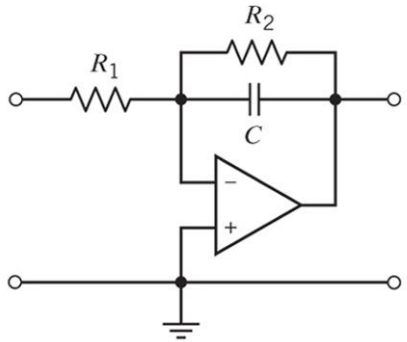
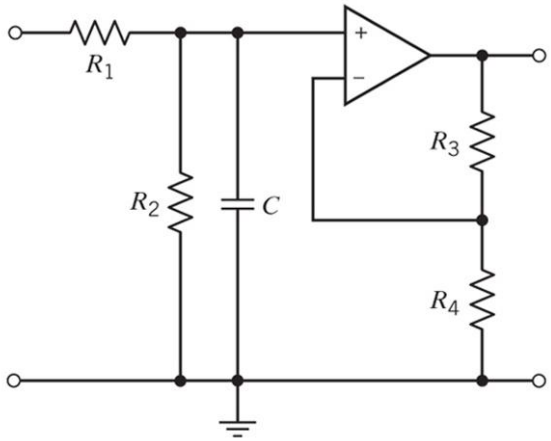
- Design the circuit to specify the following specification
  - The low-frequency gain is 0.1.
  - The high-frequency gain is 1.
  - Corner frequencies lie in the range of 100 Hz to 2000 Hz.



Example 13.3-7

- Referring the table, design a circuit that has the asymptotic magnitude Bode plot shown.



FIRST-ORDER LOW-PASS FILTER CIRCUIT	DESIGN EQUATIONS
	$H_0 = -\frac{R_2}{R_1}$ $\omega_0 = \frac{1}{R_2 C}$
	$H_0 = \frac{R_2}{R_1 + R_2}\left(1 + \frac{R_3}{R_4}\right)$ $\omega_0 = \frac{R_1 + R_2}{R_1 R_2 C}$

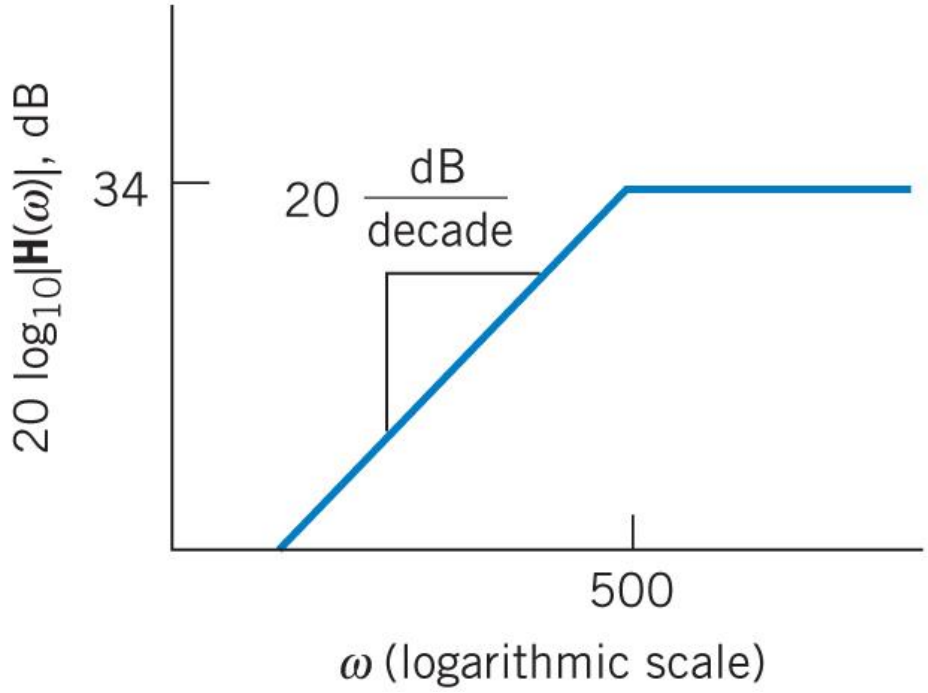
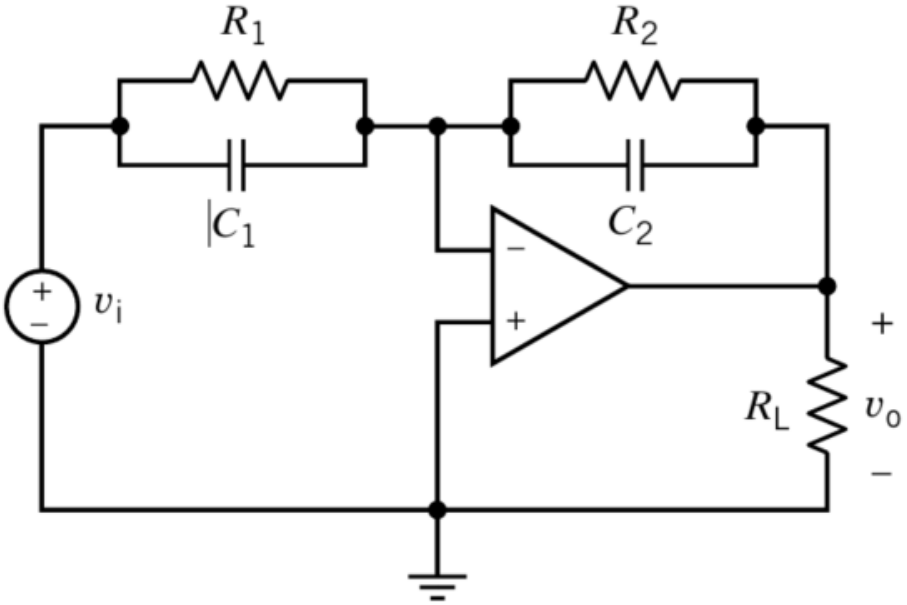

20 log10|H(ω)|, dB
34
20 dB/decade
500
ω (logarithmic scale)

Table 13.3-2



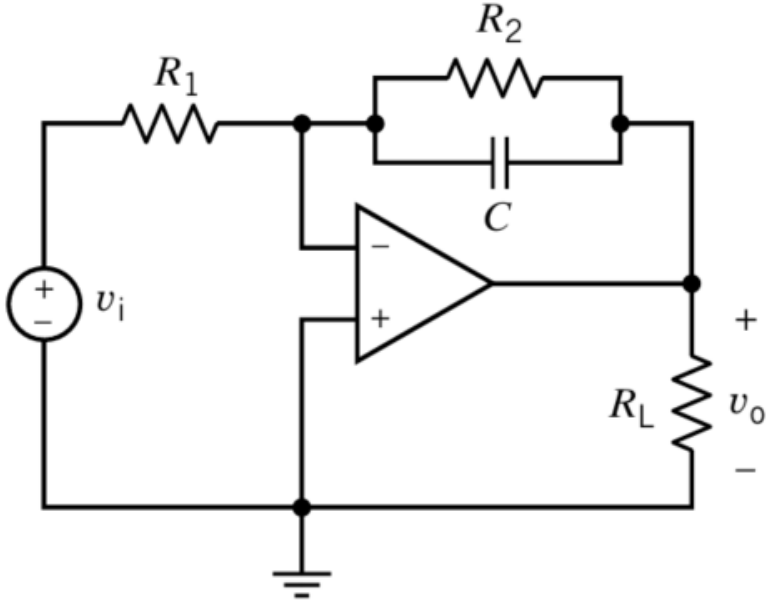
$$\mathbf{H}(\omega) = -k\,\frac{1 + j\,\frac{\omega}{z}}{1 + j\,\frac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1 R_1}$$

$$p = \frac{1}{C_2 R_2}$$



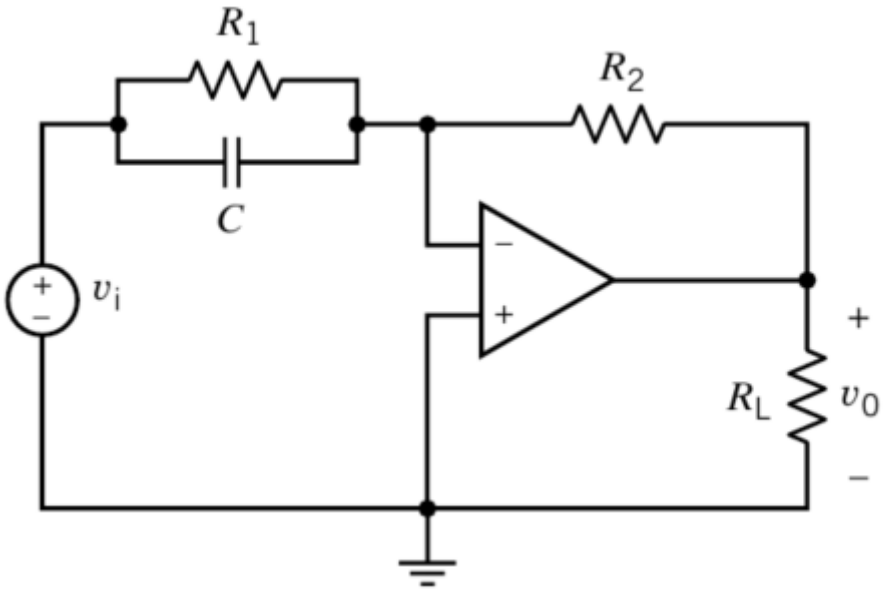
$$\mathbf{H}(\omega) = -\,\frac{k}{1 + j\,\frac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$p = \frac{1}{CR_2}$$

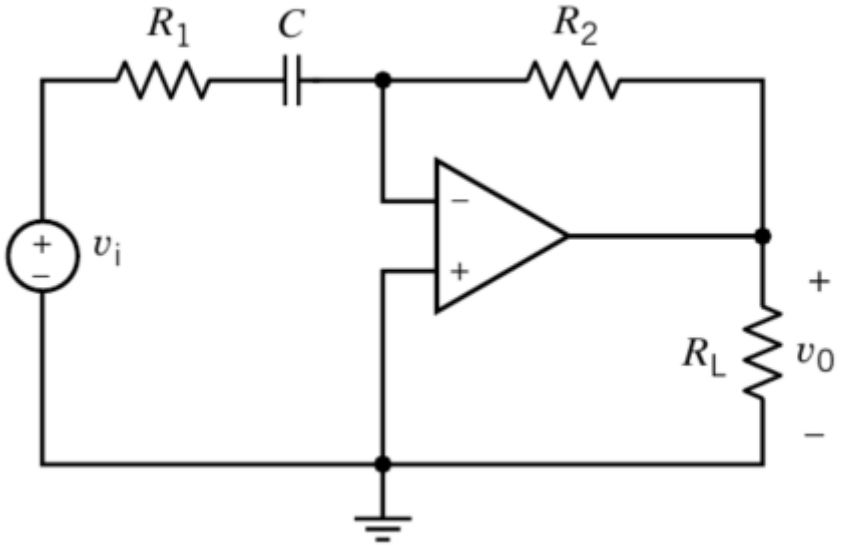
Table 13.3-2



$$\mathbf{H}(\omega) = -k\left(1 + j\frac{\omega}{z}\right)$$

where $k = \dfrac{R_2}{R_1}$

$z = \dfrac{1}{CR_1}$

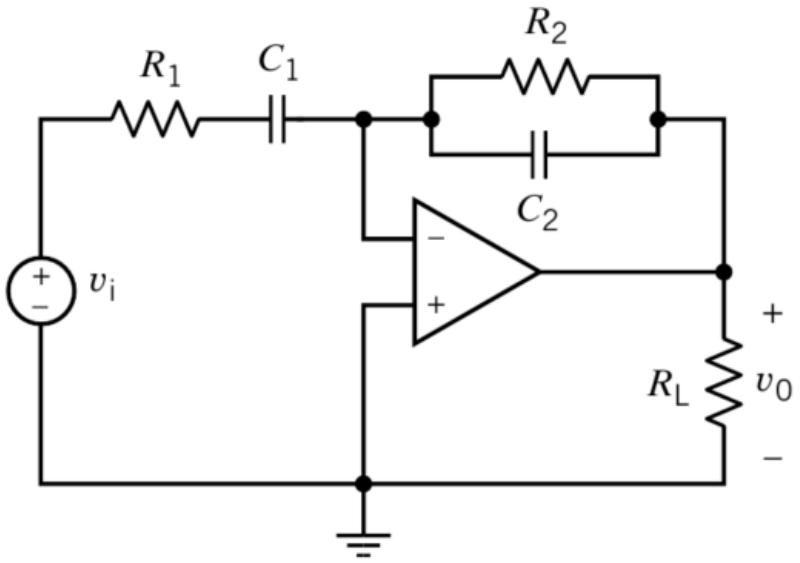


$$\mathbf{H}(\omega) = -k\,\frac{j\omega}{1 + j\frac{\omega}{p}}$$

where $k = R_2C$

$p = \dfrac{1}{CR_1}$

Table 13.3-2



$$\mathbf{H}(\omega) = -\frac{k(j\omega)}{\left(1 + j\frac{\omega}{p_1}\right)\left(1 + j\frac{\omega}{p_2}\right)}$$

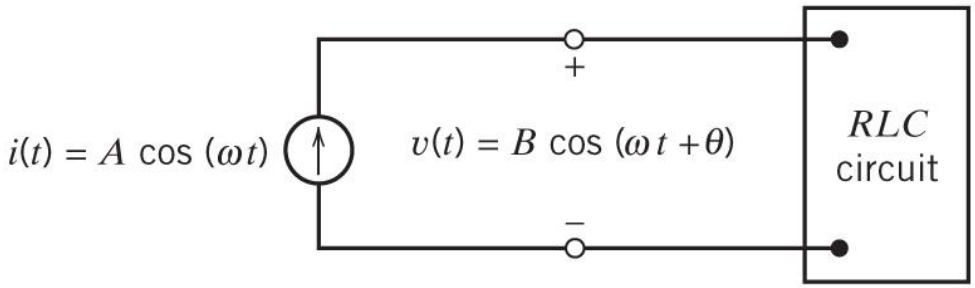
where $k = C_1 R_2$

$$p_1 = \frac{1}{C_1 R_1}$$

$$p_2 = \frac{1}{C_2 R_2}$$

Resonant Circuits

- Consider the following situation

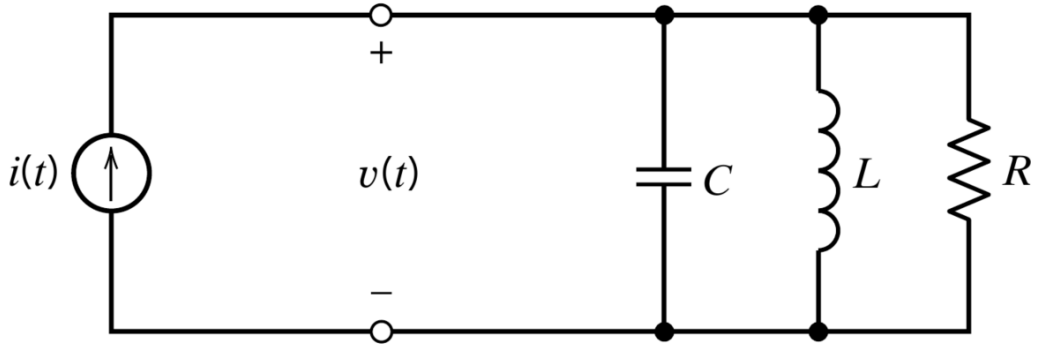


A, A	ω, rad/s	B, V	θ
0.002	200	6.6	48°
0.002	220	8.4	33°
0.002	250	10.0	0°
0.002	270	9.3	–21°
0.002	300	7.4	–43°

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{A\angle\theta}{B\angle 0^\circ}$$

- $\theta > 0$: inductive reactance, $\theta < 0$: capacitive reactance
- $\theta = 0$
  - Purely resistive
  - Magnitude of Z is max
  - ω making this condition is the resonant frequency ω_0.

Parallel Resonant Circuit



- Equivalent impedance:

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}} \Big/ -\tan^{-1} R\left(\omega C - \frac{1}{\omega L}\right)$$

- The reactance becomes zero when

$$\omega C - \frac{1}{\omega L} = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

- The impedance can be rewritten as:

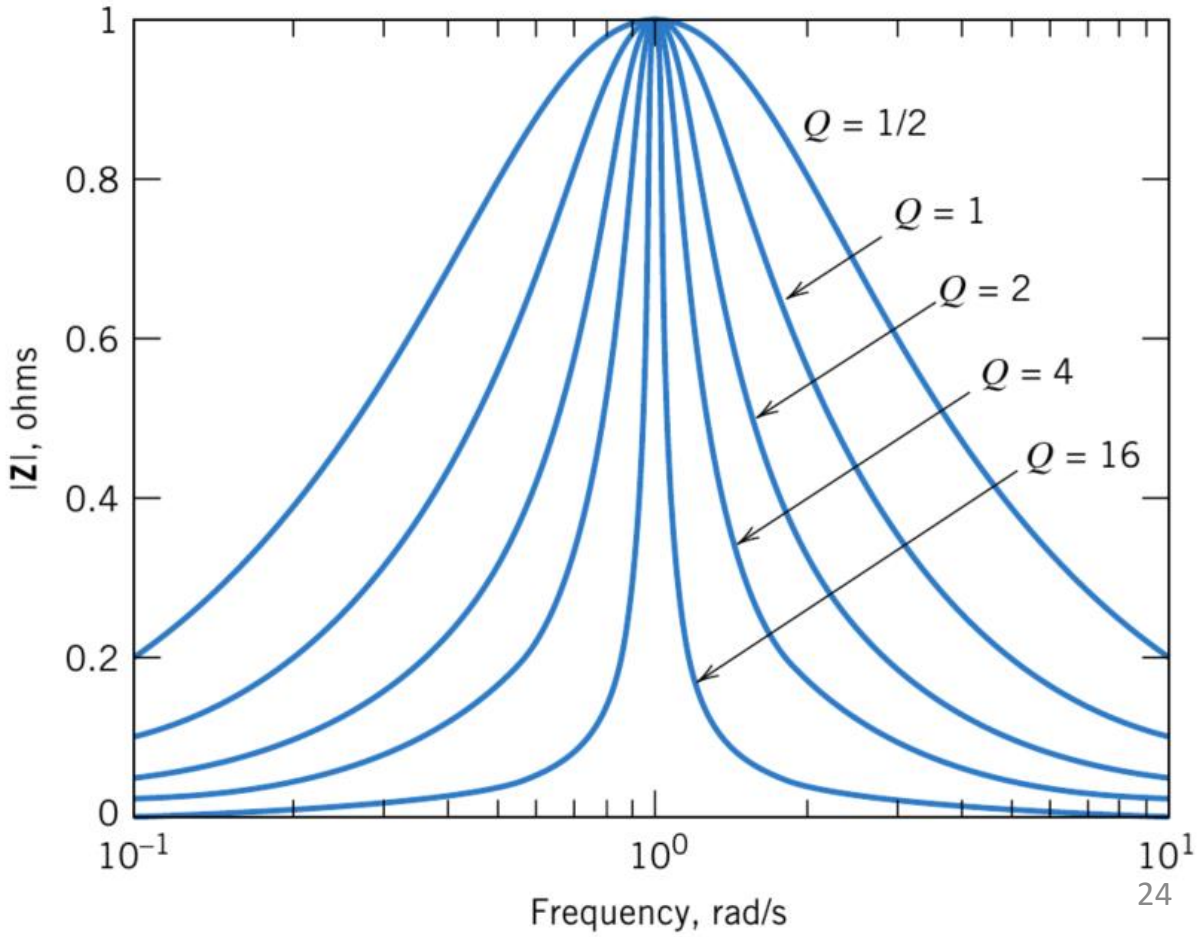
$$\mathbf{Z} = \frac{k}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$k = R, \quad Q = R\sqrt{\frac{C}{L}}, \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Key parameters of the Resonant Circuit

- Recall that

- k : Impedance when $\omega = \omega_0$, ➔ k is the max value of the impedance.
- Q : Quality factor



Bandwidth of the Resonant Circuit

- There are two frequencies ω_1 and ω_2 such that

$$|\mathbf{Z}(\omega)| = \frac{1}{\sqrt{2}}|\mathbf{Z}(\omega_0)| = \frac{k}{\sqrt{2}}$$

where $\omega_1 < \omega_0$ and $\omega_2 > \omega_0$

- Then BW can be defined as BW = $\omega_2 - \omega_1$.
- We can derive ω_1 or ω_2 with

$$\frac{k}{\sqrt{2}} = \frac{k}{\sqrt{1 + Q^2(\omega/\omega_0 - \omega_0/\omega)^2}} \quad \Rightarrow \quad \pm 1 = Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

- Among the four possible two solutions, picking two positive solutions,

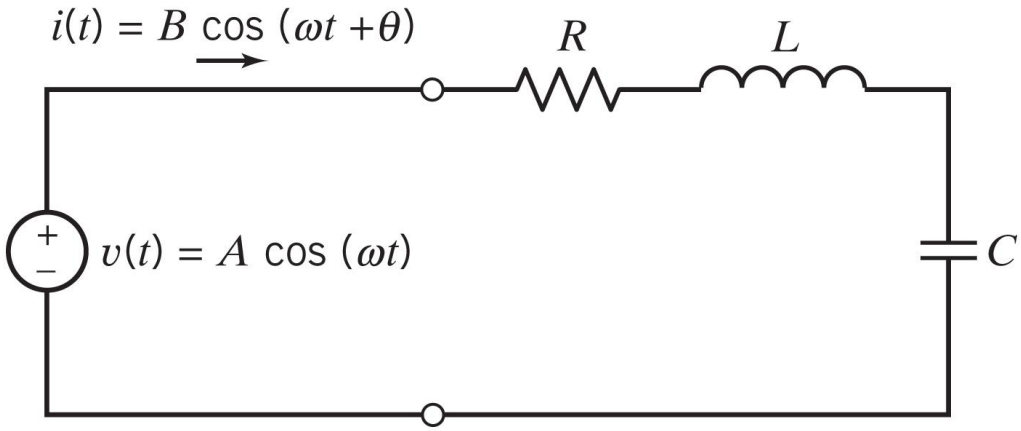
$$\omega_1 = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^2} \quad \text{and} \quad \omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^2}$$

- Then,

$$BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

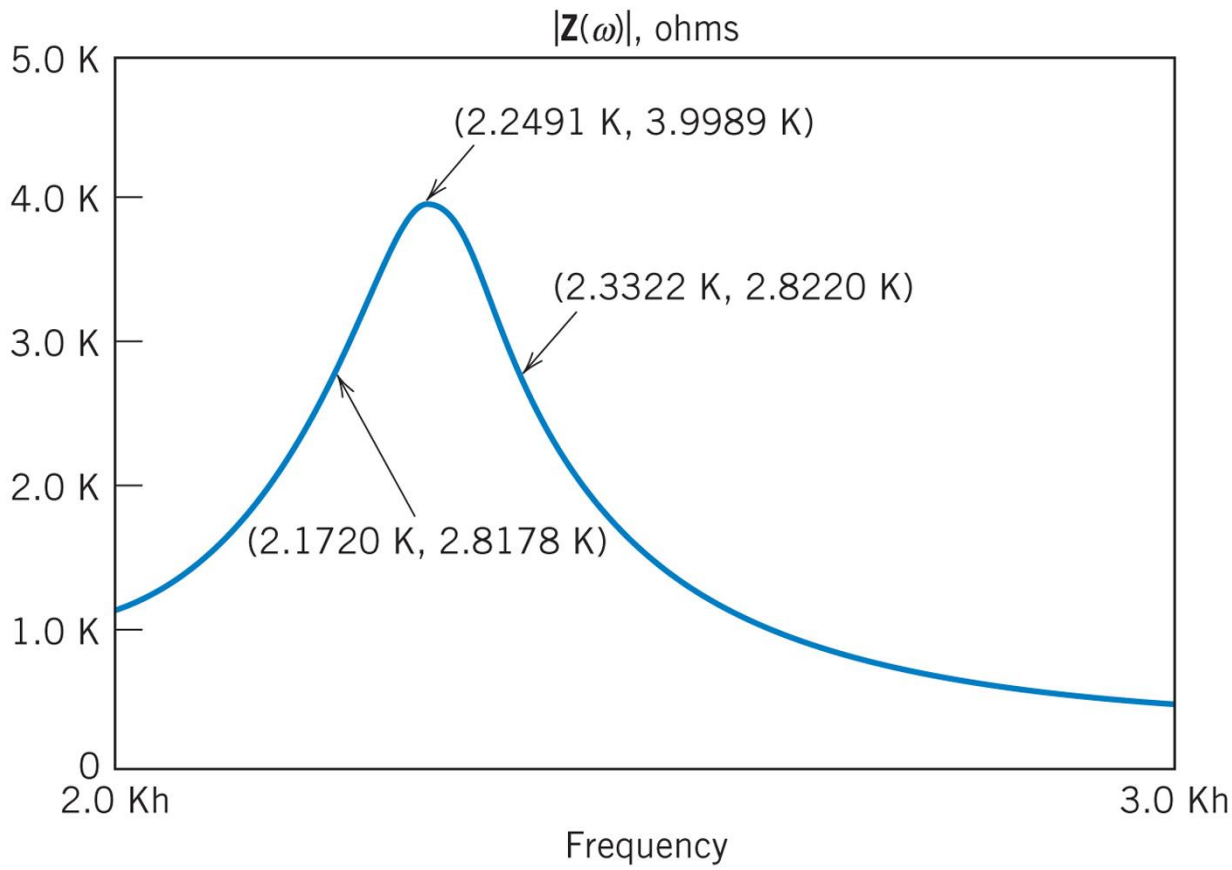
Example 13.4-1

- Determine the relationship between parameters k, Q, and ω_0 and the element values R, L, and C.



Example 13.4-2

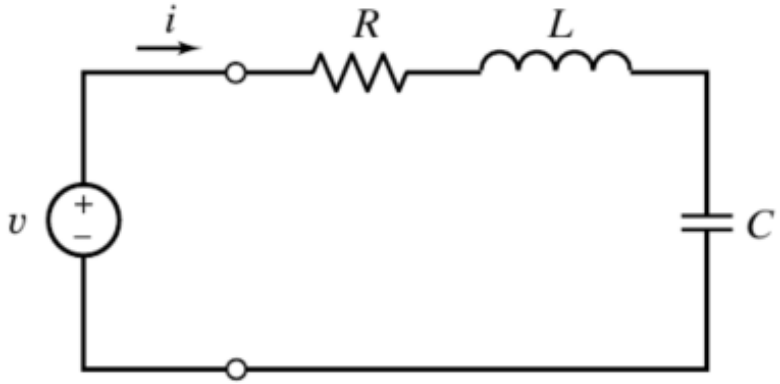
- Determine k, Q, and ω_0.



Example 13.4-3

- Design a parallel resonant circuit that has k = 4000 Ω, Q = 14, and ω_0 = 14,130 rad/s.

Table 13.4-1



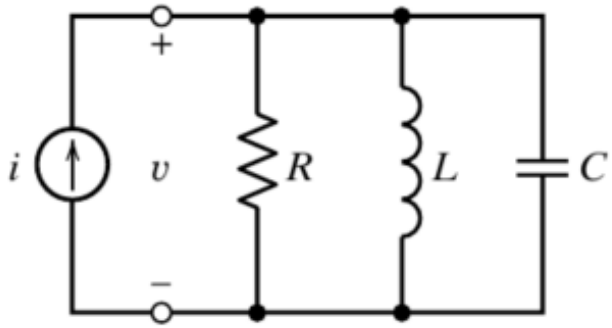
$$\mathbf{Y} = \frac{k}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$k = \frac{1}{R}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$BW = \frac{R}{L}$$



$$\mathbf{Z} = \frac{k}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$k = R$$

$$Q = R\sqrt{\frac{C}{L}}$$

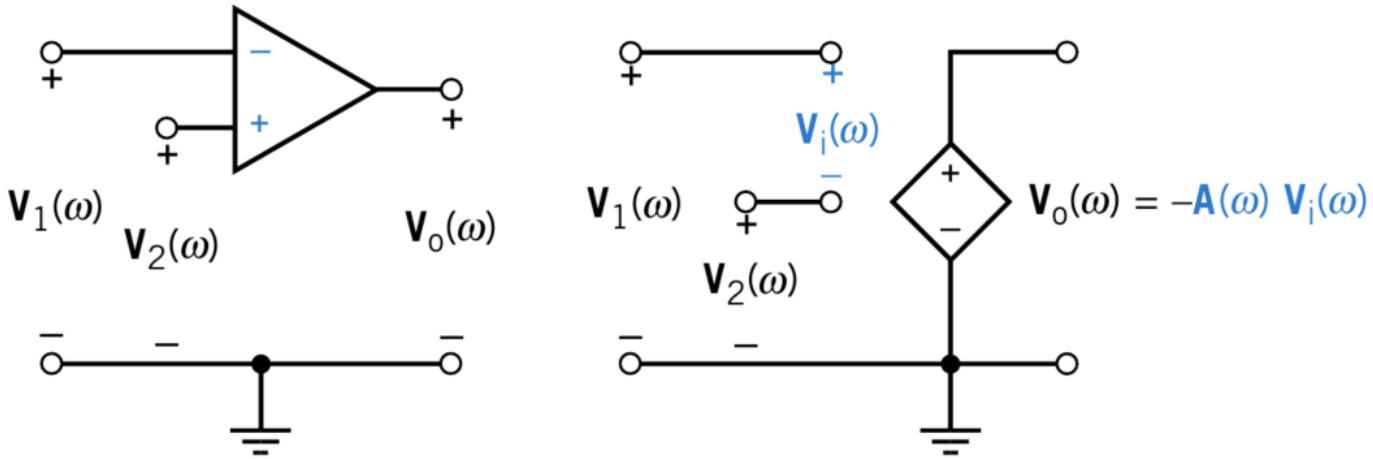
$$BW = \frac{1}{RC}$$

Frequency Response of Op-Amp Circuits

- The gain of an op amp is not infinite; rather, it is finite and decreases with frequency. We can write as:

$$\mathbf{A}(\omega) = \frac{A_o}{1 + j\omega/\omega_1}$$

- Normally, the dc gain $A_o > 10^4$ and $\omega_1 < 100$ rad/s.



Example 13.5-1

- R_1 = 10kΩ, R_2 = 90kΩ, $A_o = 10^5$, and ω_1 = 10 rad/s.
- Find the Bode plot for A(ω) and the network function of V_o/V_s.

