Chap. 14

The Laplace Transform (2)

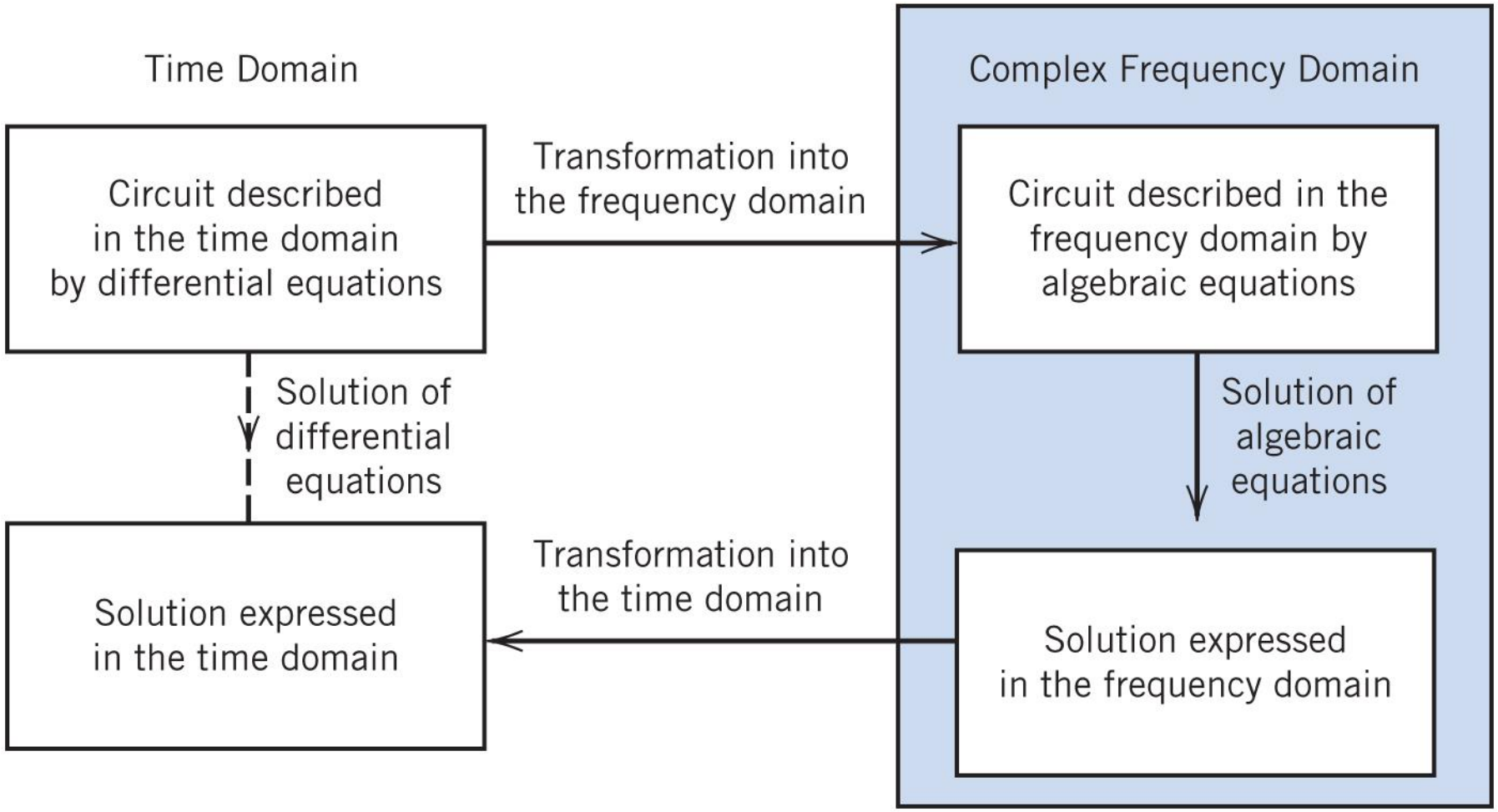
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Laplace Transform for Circuit Analysis

- ✓ 14.6 Solution of Differential Equations Describing a Circuit
- ✓ 14.7 Circuit Analysis Using Impedance and Initial Conditions
- ✓ 14.8 Transfer Function and Impedance
- ✓ 14.10 Stability

Revisiting Transform Method



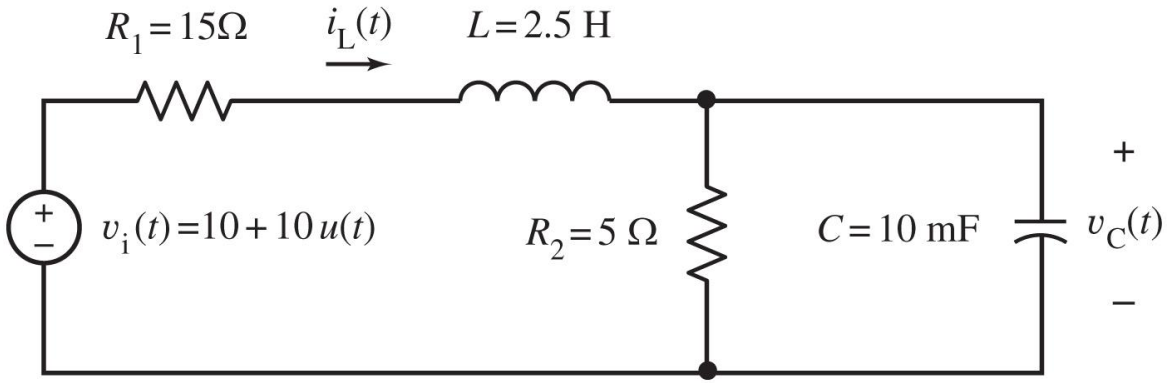
What We have Learned As RLC Circuit Analysis Method

- Chap 9: Directly solving thorough differential equation
- Chap 10: Phasor for AC circuit
  - Sinusoidal input & steady state
  - Still needs direct method for handling transient response
- Chap 13: Frequency response (Generalizing frequency)
  - Limited to steady state response
- Chap 14: What can we do with Laplace transform?
  - Laplace transform can simplify differential equation solving procedure
  - We will not limit the signal as the sinusoidal function.

Solving Diff. Eq Describing a Circuit

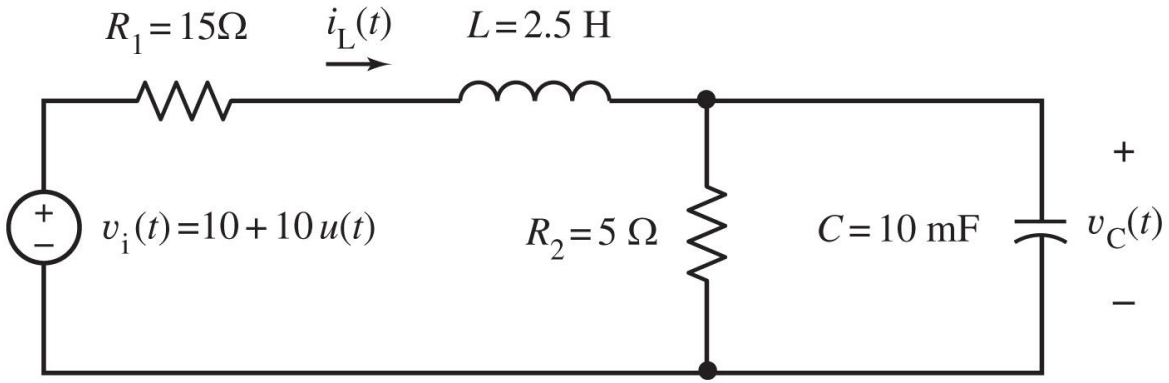
1. Use Kirchhoff's laws and the element equations to represent the circuit by a differential equation or set of differential equations.
2. Transform each differential equation into an algebraic equation by taking the Laplace transform of both sides of the equation.
3. Solve the algebraic equations to obtain the Laplace transform of the output of the circuit.
4. Take the inverse Laplace transform to obtain the circuit output itself.

Example 14.6-1

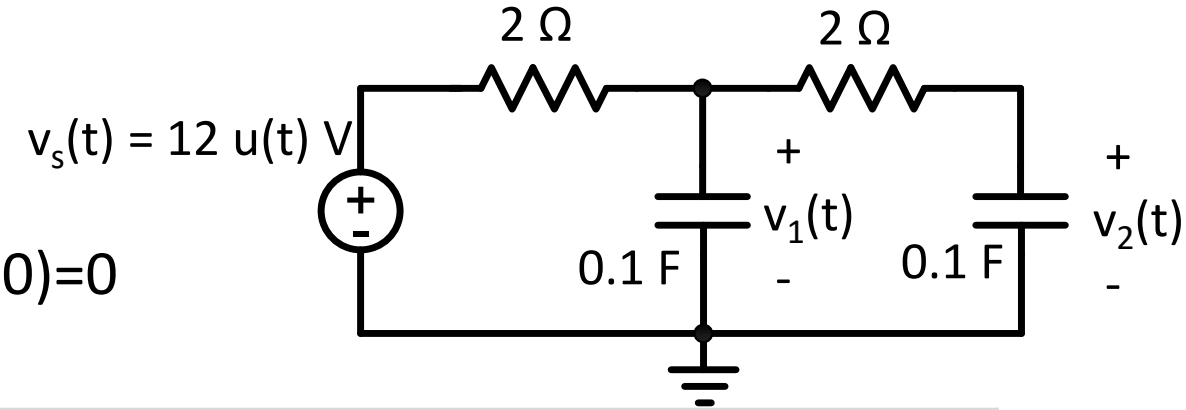


- Find $v_C(t)$ when $i_L(0\text{-}) = 0.5$ A and $v_C(0\text{-}) = 2.5$ V.

Example 14.6-1

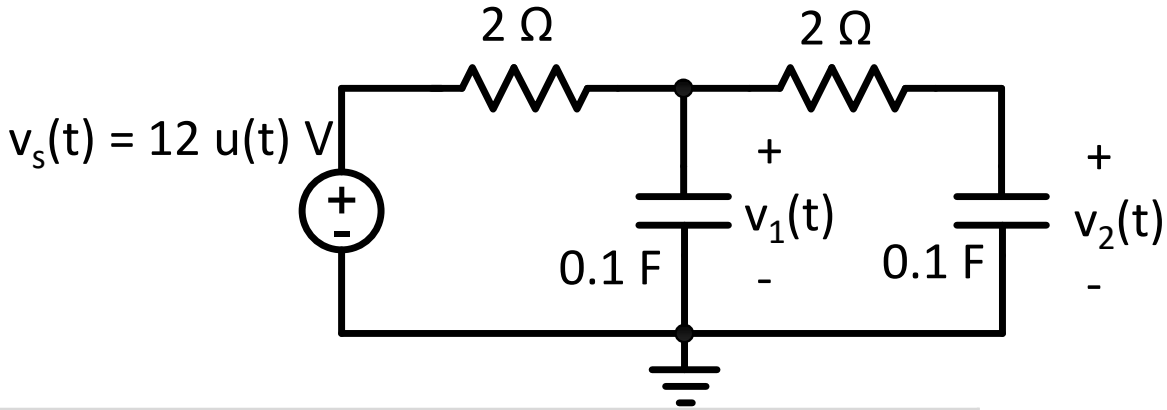


Example 14.6-2



- Find $v_2(t)$ when $v_1(0) = 0$ and $v_2(0)=0$

Example 14.6-2



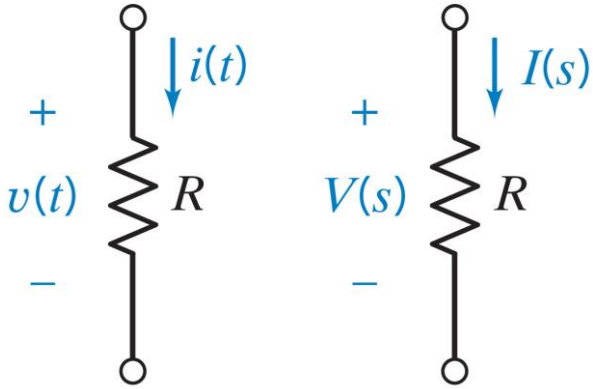
Circuit Analysis Using Impedance and Initial Conditions (Resistor)

- In time domain,

$$v(t) = i(t)R$$

- With Laplace Transform, in s domain

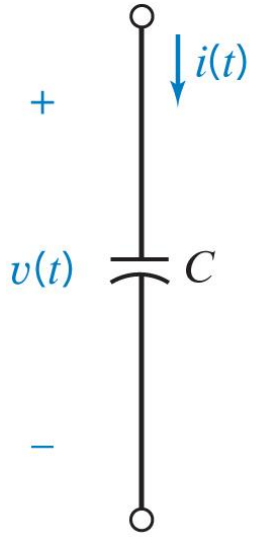
$$V(s) = I(s)R$$



Circuit Analysis Using Impedance and Initial Conditions (Capacitor)

- In time domain,

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0)$$

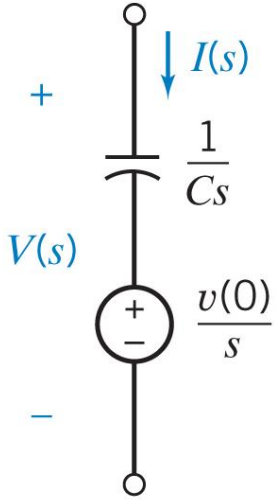


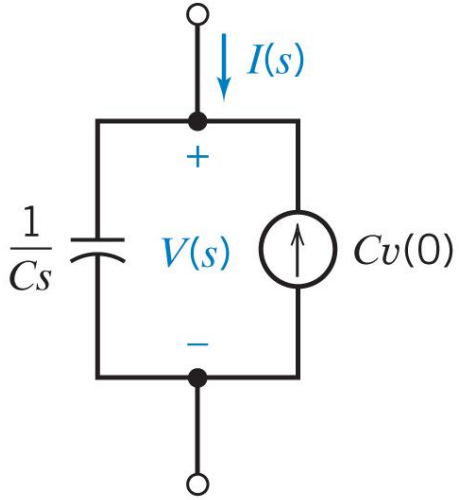
- With Laplace Transform, in s domain

$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s}$$

which leads to

$$I(s) = CsV(s) - Cv(0)$$





Integration in Time-Domain

Time domain Frequency domain

$$\int_{0^-}^{t} f(x)dx \quad \leftrightarrow \quad \frac{F(s)}{s}$$

$$\int_{0^-}^{\infty}\left(\int_{0^-}^{t} f(x)dx\right)e^{-st}dt$$

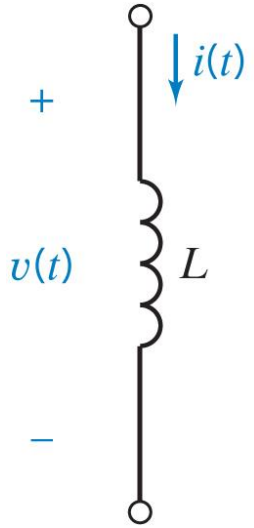
$$= -\left(\int_{0^-}^{t} f(x)dx\right)\frac{1}{s}e^{-st}\Big]_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t)\left(-\frac{1}{s}\right)e^{-st}dt$$

$$= \frac{1}{s}\int_{0^-}^{\infty} f(t)e^{-st}dt$$

Circuit Analysis Using Impedance and Initial Conditions (Inductor)

- In time domain,

$$v(t) = L\frac{d}{dt}i(t)$$

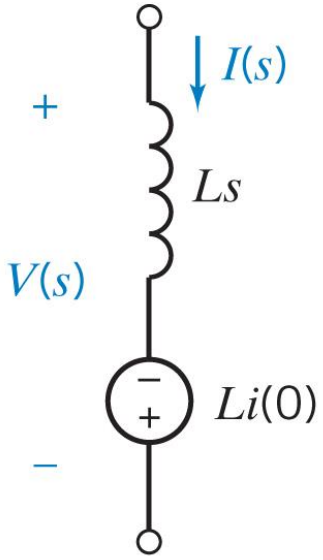


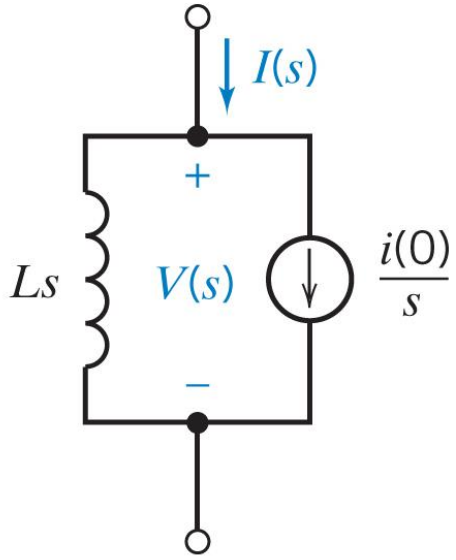
- With Laplace Transform, in s domain

$$V(s) = LsI(s) - Li(0)$$

which leads to

$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0)}{s}$$



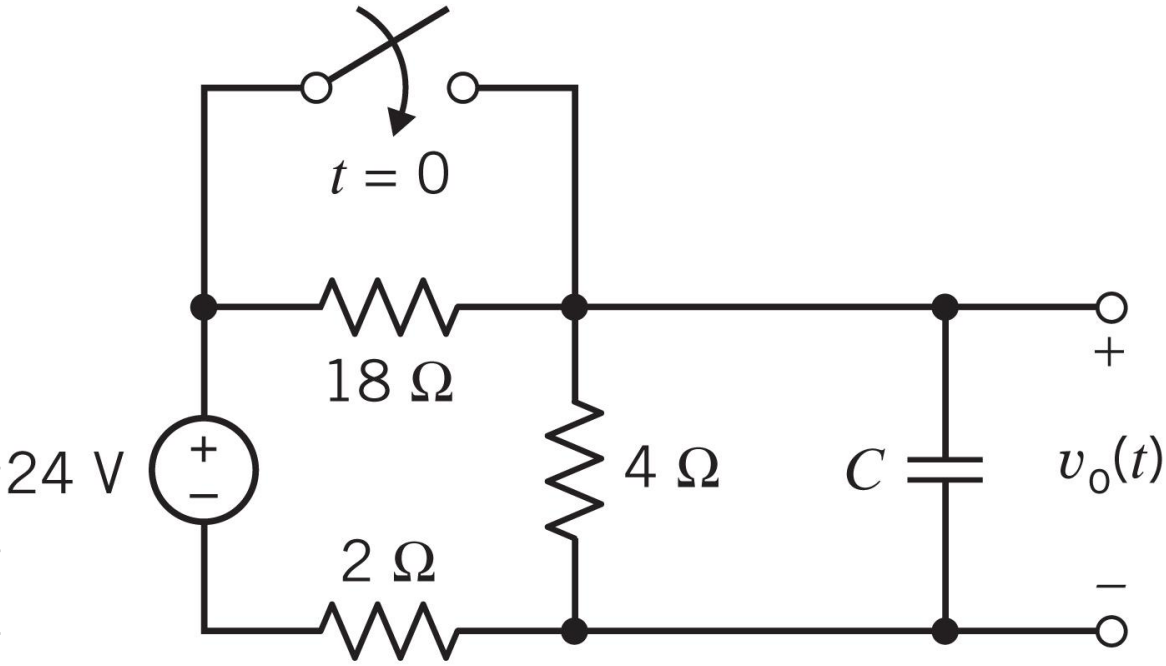


Example 14.7-1

- The output is given by

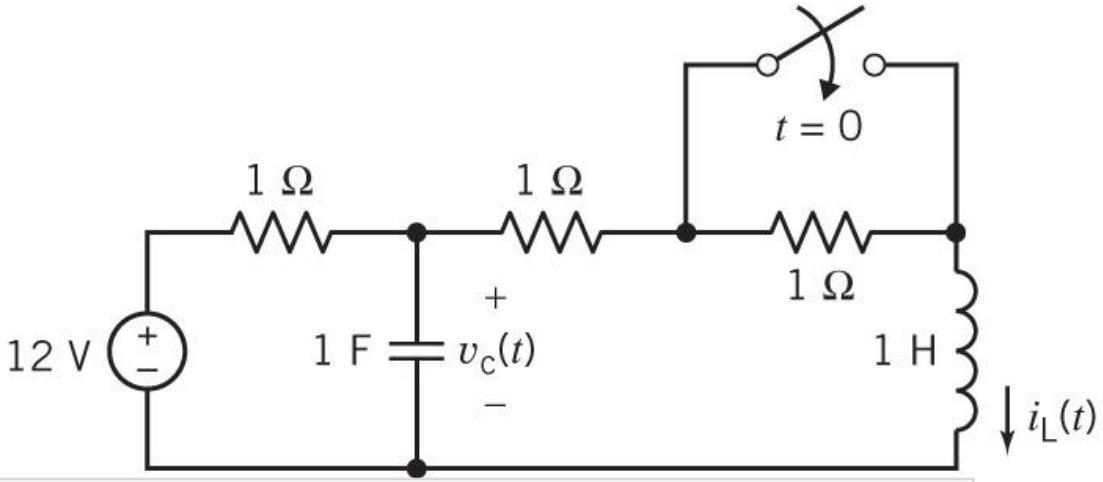
$$v_o(t) = 16 - 12e^{-0.6t}\ \text{V}$$

- Determine the value of C.

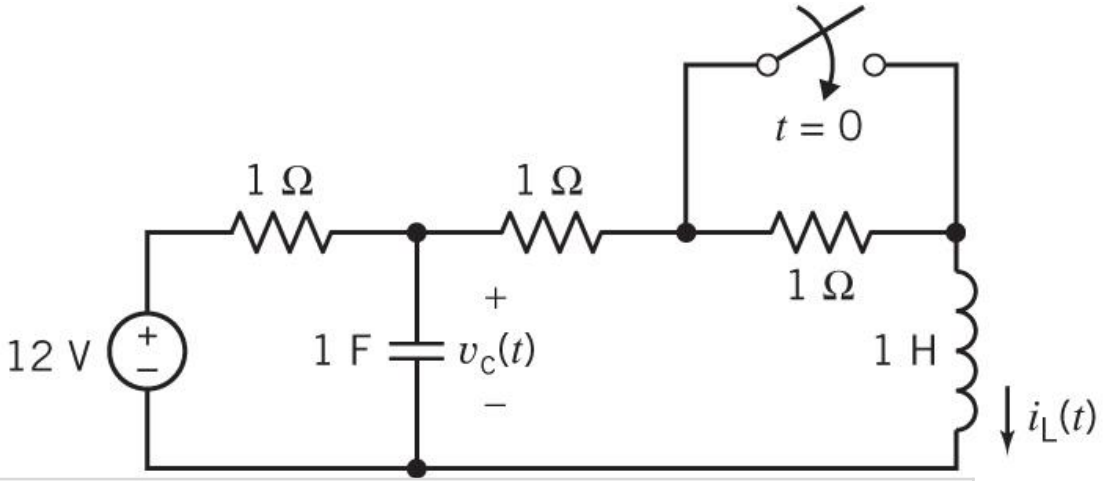


Example 14.7-3

- Determine $i_L(t)$

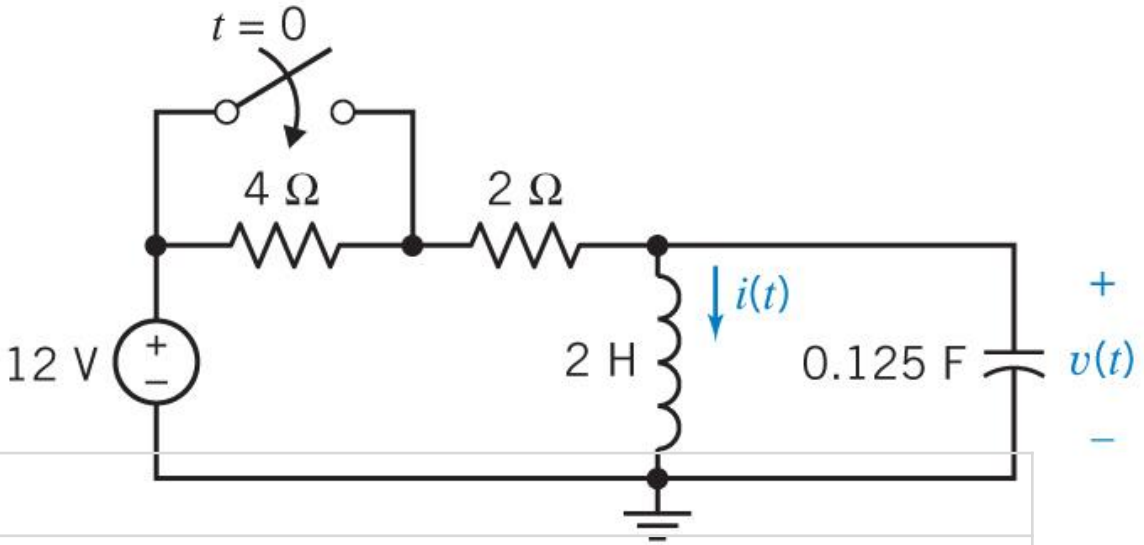


Example 14.7-3



Example 14.7-4

- Determine v(t).



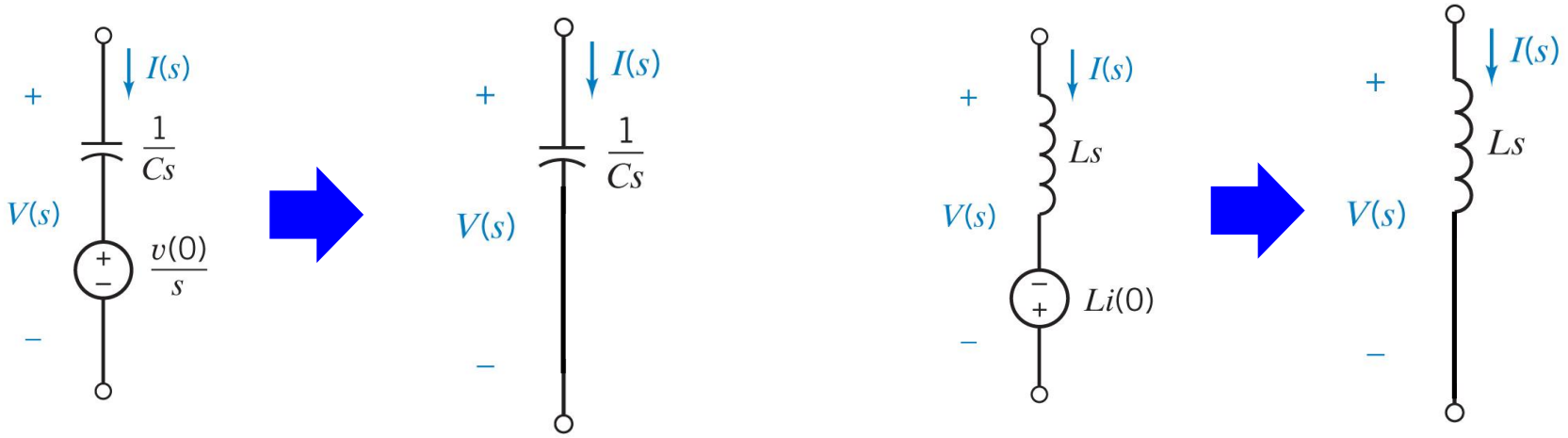
Zero Initial Condition and Impedance

- When initial voltage of a capacitor is zero, v(0)=0, the frequency-domain representation of a capacitor is reduced a single element.
- That is, the impedance of the capacitance can be defined as

$$Z_C(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs}$$

- Similarly an inductor with zero initial condition, i(0)=0, have the impedance of

$$Z_L(s) = Ls$$



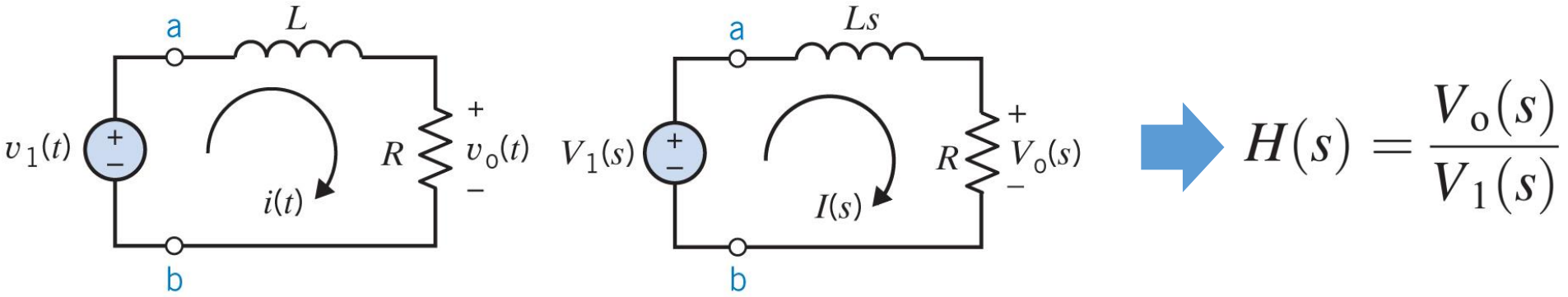
Transfer Function

- Transfer function:

$$\boldsymbol{H(s) = \frac{Output(s)}{Input(s)}}$$

  when the initial conditions are zero (shortly, **zero-state response**).

- Example



$$H(s) = \frac{V_o(s)}{V_1(s)}$$

- Output in s-domain can be obtained by

$$V_o(s) = H(s)V_1(s)$$

Step Response & Impulse Response

- When the input is a unit step function,

$$V_1(s) = \mathscr{L}[u(t)] = \frac{1}{s} \quad \Rightarrow \quad V_o(s) = \frac{H(s)}{s}$$

- In this case, the zero-state response is called the step response:

$$\text{step response} = \mathscr{L}^{-1}\left[\frac{H(s)}{s}\right]$$

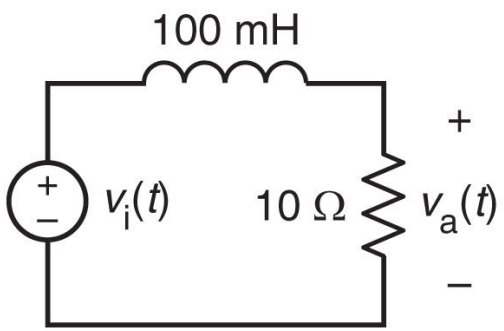
- When the input is an impulse function,

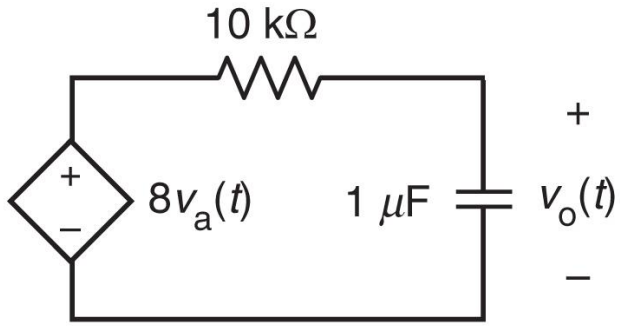
$$V_1(s) = \mathscr{L}[\delta(t)] = 1 \quad \Rightarrow \quad V_o(s) = H(s)$$

- In this case, the zero-state response is called the impulse response:

$$\text{impulse response} = \mathscr{L}^{-1}[H(s)]$$

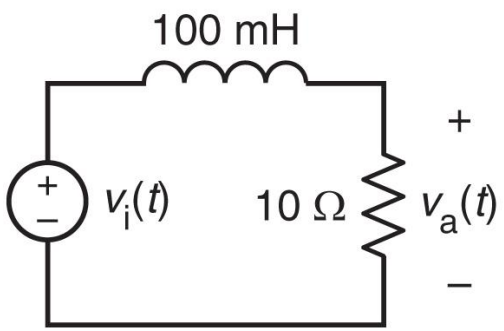
Example 14.8-1

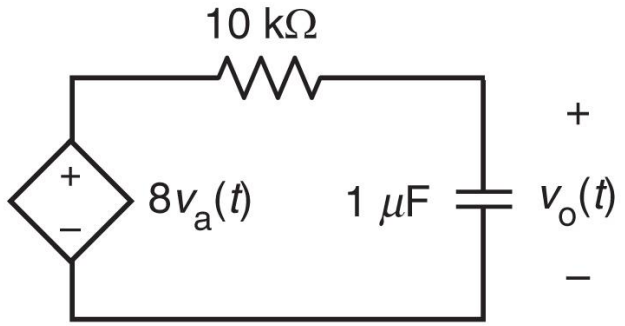




- Determine the step response.

Example 14.8-1

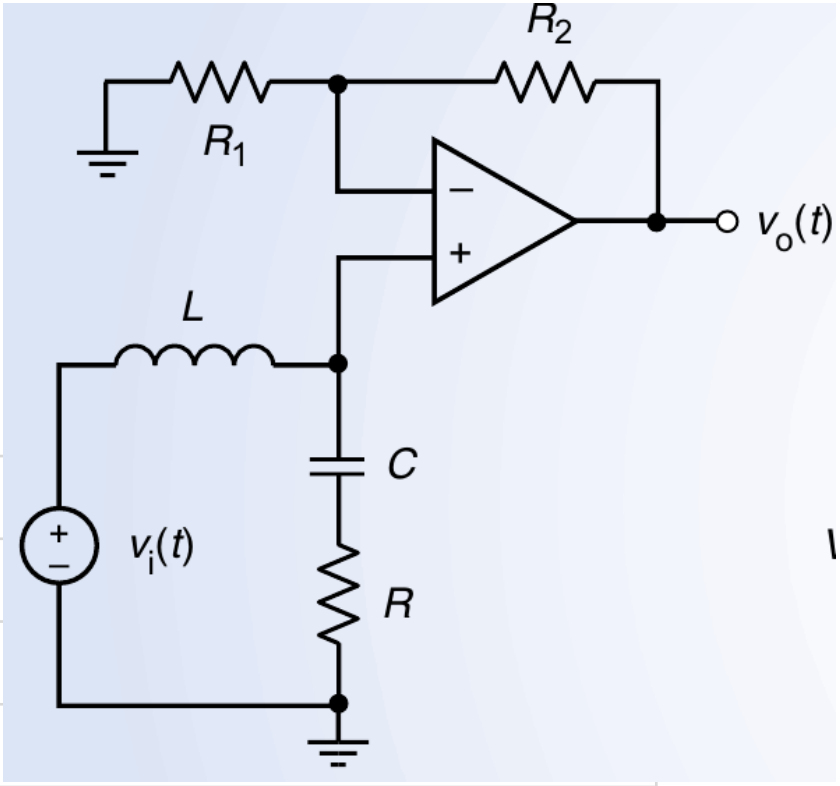


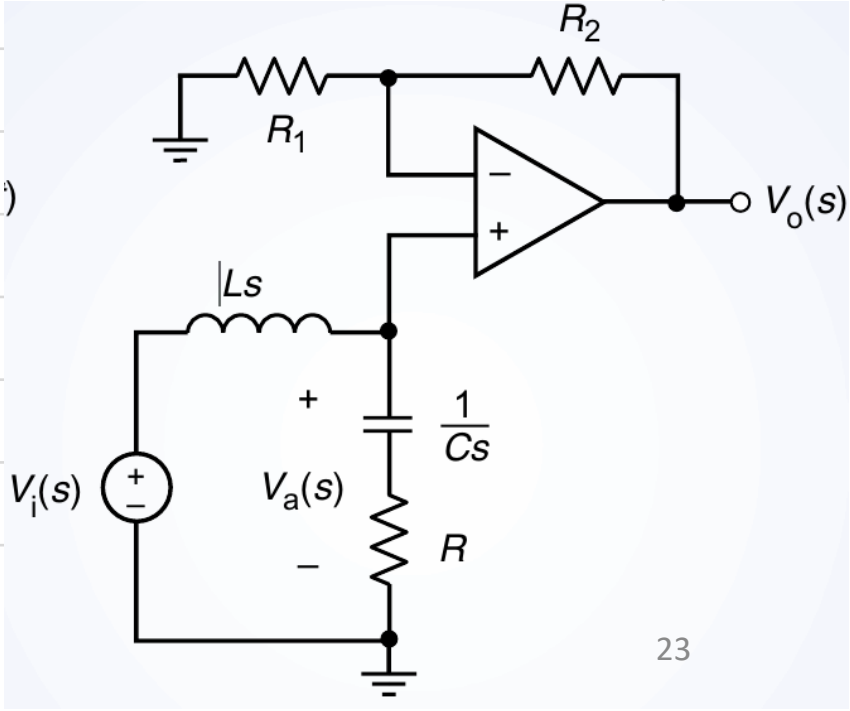


Example 14.8-2

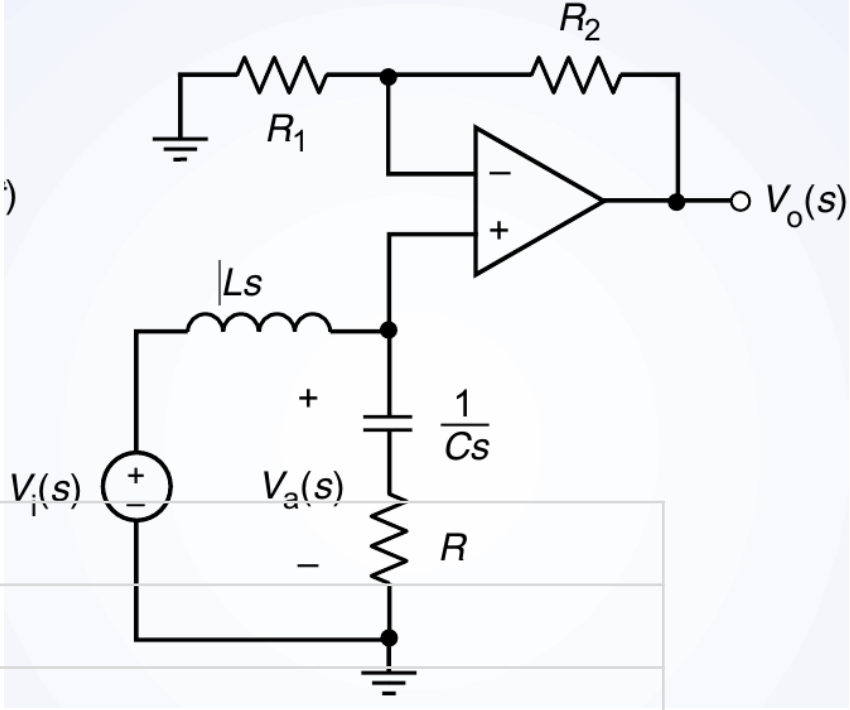
- Design the circuit to have the step response

$v_o(t) = [4 - e^{-2t}\{ 4\cos(4t) - 2\sin(4t) \}]u(t)$

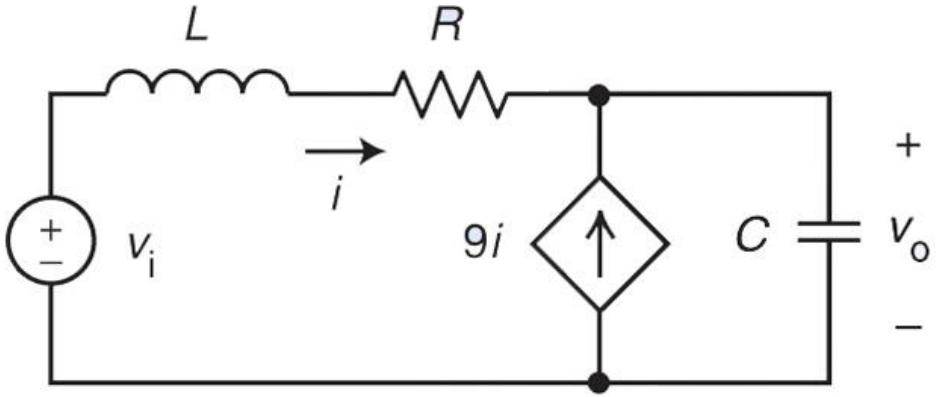




Example 14.8-2

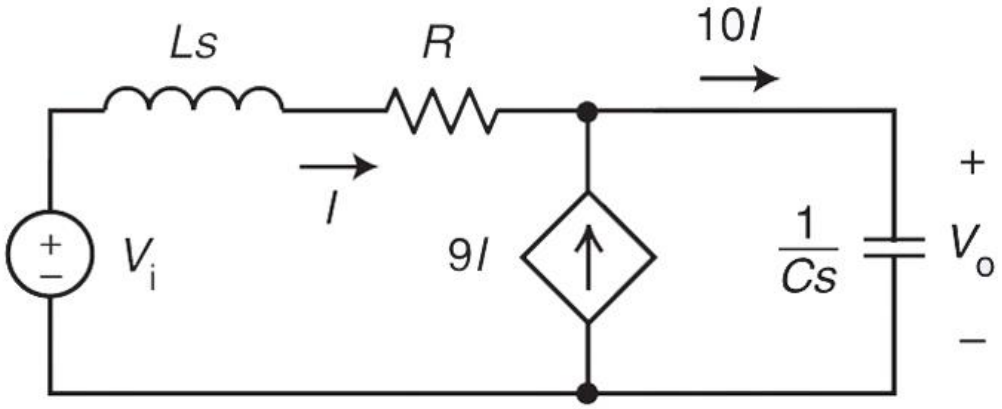


Example 14.8-3

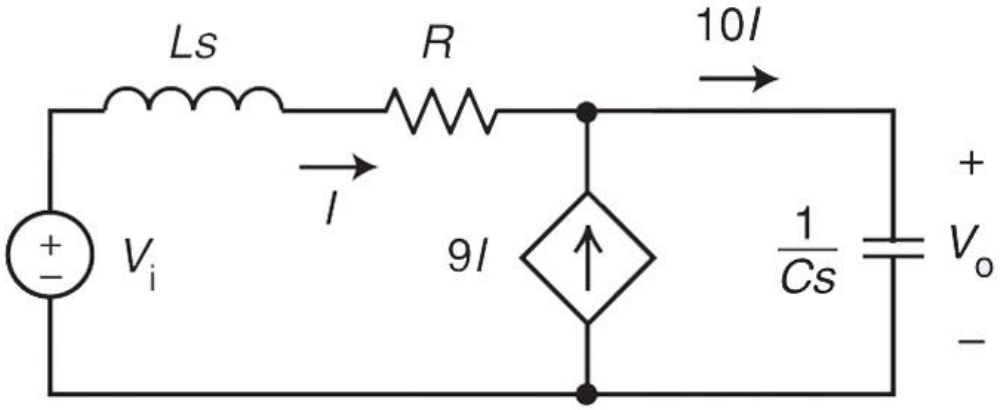


- Design the circuit for the step response

$$v_o(t) = [\ 1 - (10^4 t + 1)e^{-10{,}000t}\ ]u(t)$$



Example 14.8-3



Stability

- A circuit is stable when the response to a bounded input signal is a bounded output signal. A circuit that is not stable is said to be unstable.
- Consider the following transfer function,

$$H(s) = \frac{R_1}{s - p_1} + \frac{R_2}{s - p_2} + \frac{R_3}{s - p_3} + k(s)$$

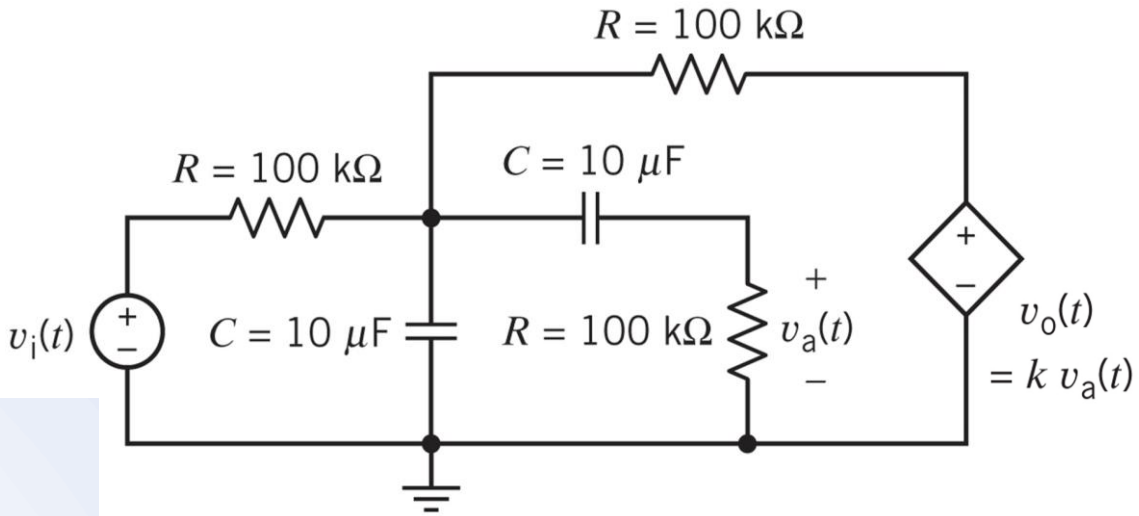
- The impulse response is

$$h(t) = \mathscr{L}^{-1}[H(s)] = \mathscr{L}^{-1}\left[\frac{N(s)}{(s - p_1)(s - p_2)\cdots(s - p_N)}\right] = \sum_{i=1}^{N} A_i e^{p_i t} u(t)$$

- A circuit is stable if, and only if, all of its poles have negative real parts.
- The relationship between the network function & transfer function?

$$\mathbf{H}(\omega) = |H(s)|_{s=j\omega}$$

Example 14.10-1



- The transfer function

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{k}{RC}s}{s^2 + \frac{4-k}{RC}s + \frac{2}{R^2C^2}} = \frac{ks}{s^2 + (4-k)s + 2}$$

1) Determine the steady-state response when $v_i(t)$ = 5cos2t V and k = 3 V/V.
2) Determine the impulse response when k = $4-2\sqrt{2}$ = 1.17 V/V
3) Determine the impulse response when k = $4+2\sqrt{2}$ = 6.83 V/V

Example 14.10-1

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