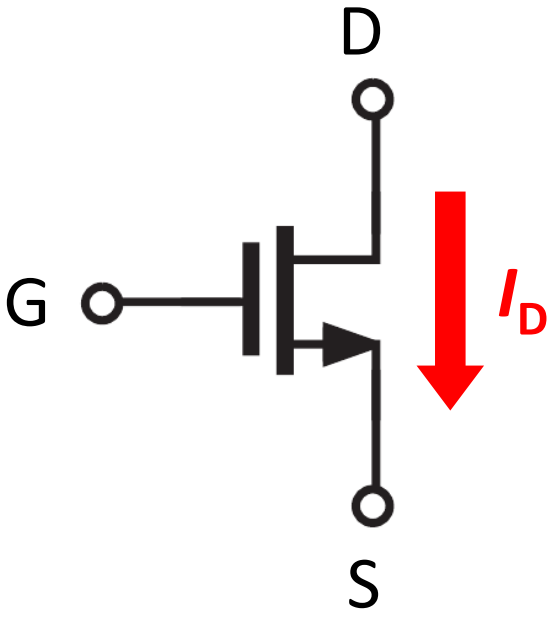
Derivation of MOSFET I-V Model

Hanwool Jeong

hwjeong@kw.ac.kr

Derivation of I-V Characteristic of MOSFET

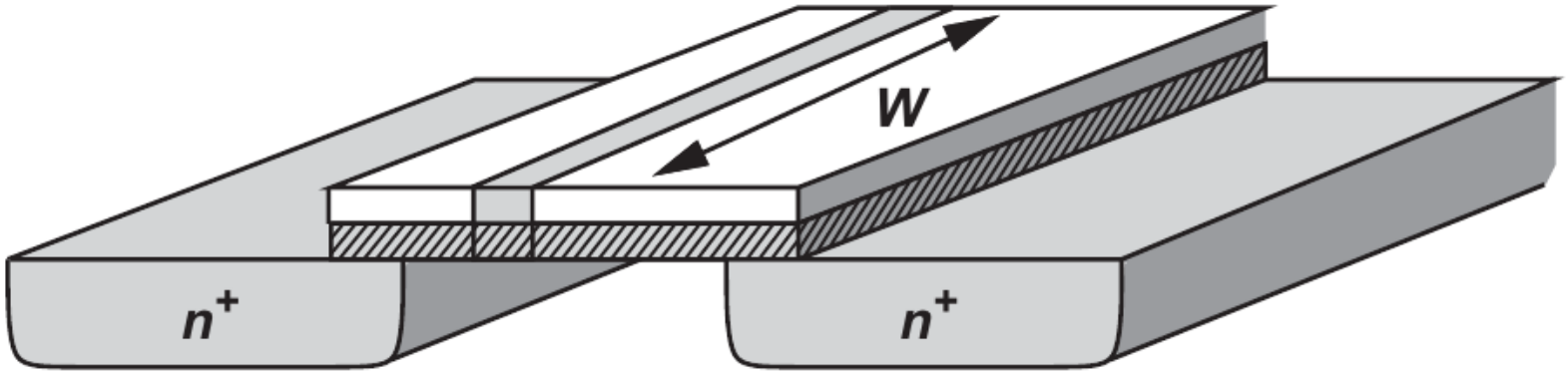
- How about I-V of MOSFET? How can we derive it?



I_D vs. V_{DS} ?
I_D vs. V_{GS} ?

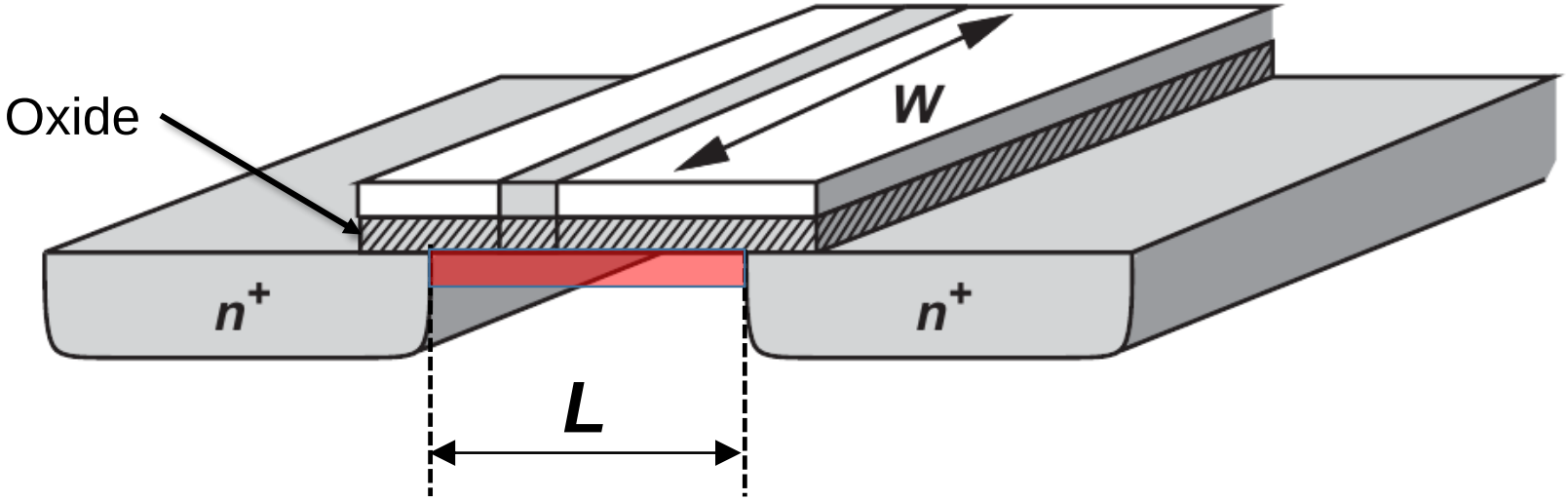
How Can We Start?

- We can start from $I = \Delta Q/\Delta t$
- Assuming **Substrate** and **Source** are grounded.
- Can you represent **ΔQ** vs. V_{GS} or V_{DS}?



Deriving Q at Oxide-Semiconductor Interface

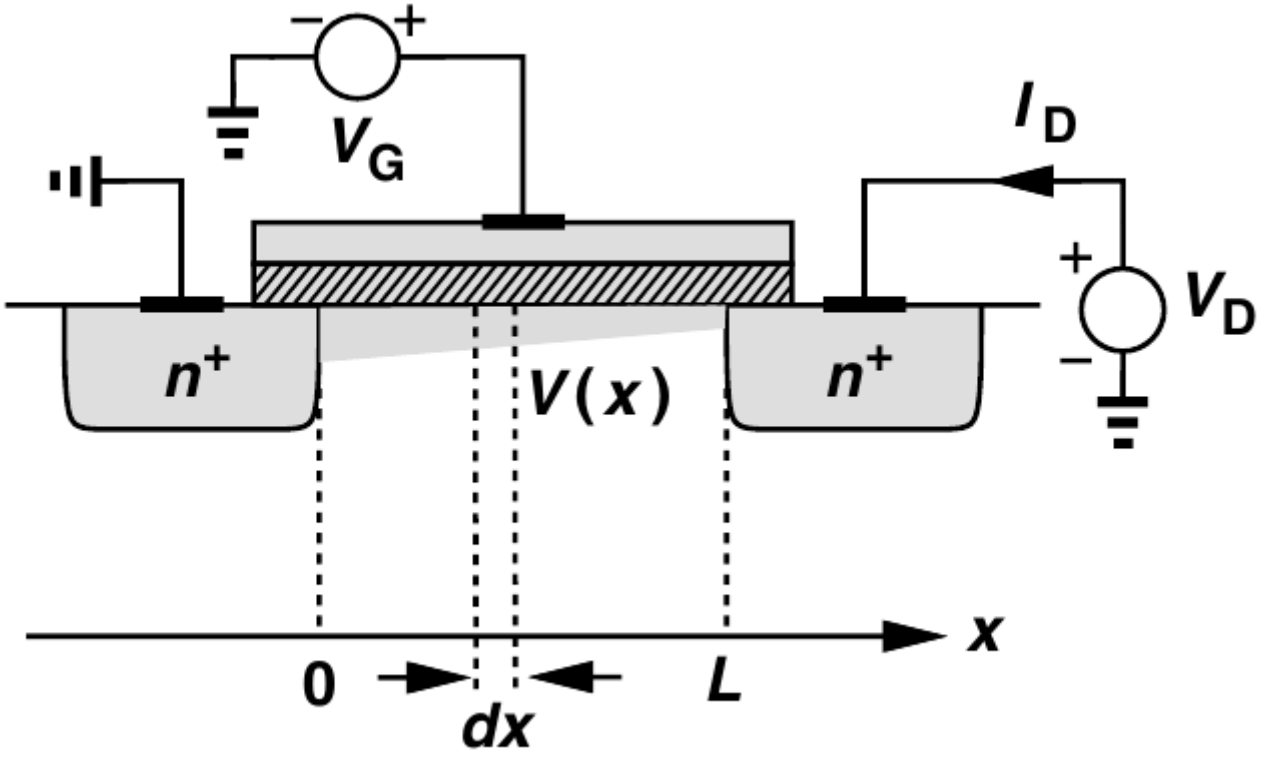
- When $V_{DS} = 0$, then $\mathbf{Q_{tot}} = CV$,
  - $V = V_{GS} - V_{TH}$
  - C is? $C = \varepsilon_{ox}\varepsilon_0 A/d = \varepsilon_{ox}\varepsilon_0 WL/t_{ox}$



- Again back to $I = \Delta Q/\Delta t$, what do we need to do?

Note that V(x) is NOT Constant Over x

- Voltage at channel is not constant over x.



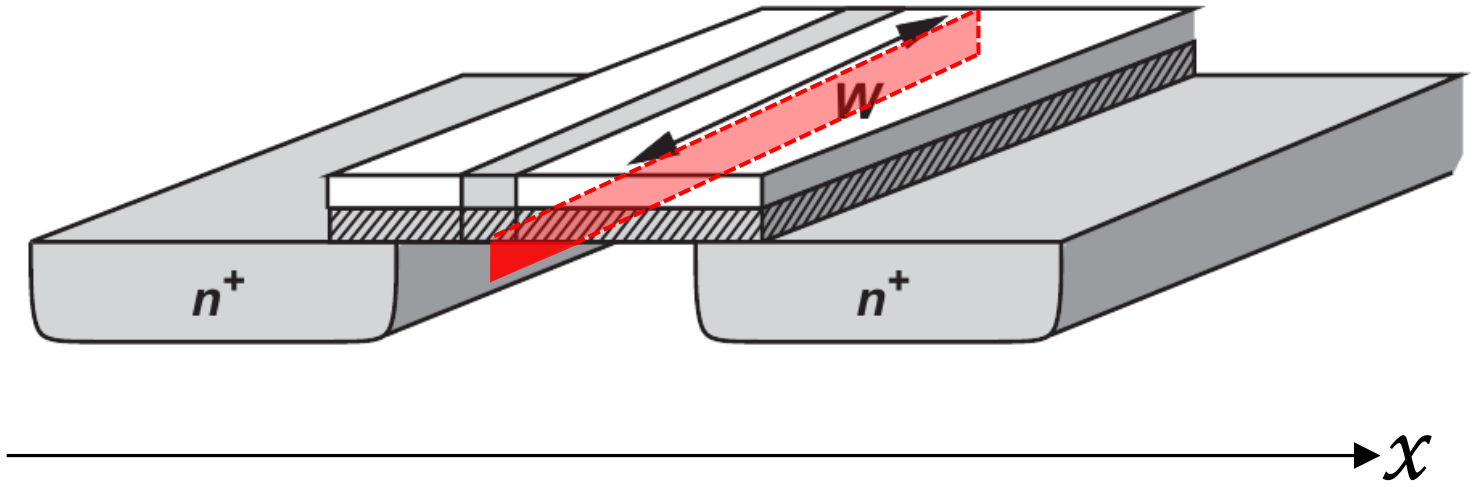
Deriving Charge Density, Q(x)

- When $V_{DS} \neq 0$, the potential of channel V(*x*) differs according to *x*.
  ➔ Charge should be defined according to x, Q(x), as the charge density
- If you integrate Q(x) respect to x, we can derive total charge.

$$Q(x) = (C/L)V = \frac{\varepsilon_{ox}\varepsilon_0 WL}{t_{ox}}\frac{1}{L}(V_{GS}-\mathbf{V(x)}-V_{TH}) = \frac{\varepsilon_{ox}\varepsilon_0 W}{t_{ox}}(V_{GS}-\mathbf{V(x)}-V_{TH})$$

$$= C_{ox}W(V_{GS}-\mathbf{V(x)}-V_{TH})$$

Defined as C_{ox}



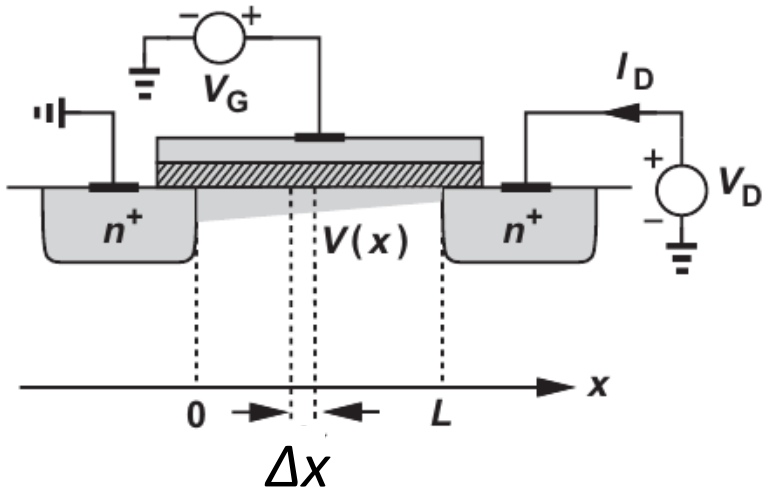
Q(x) to I_D (con't)

- Remind that I = ΔQ/Δt. We have Q(x). How does it relate to I_D?
- With small *Δx,*

$$\Delta Q = Q(x)\Delta x \rightarrow I_D = \frac{Q(x)\Delta x}{\Delta t} = Q(x)v$$

- You remember how charge moving velocity in silicon respond to voltage difference?

$$v = -\mu_n E = \mu_n \frac{dV}{dx}$$



Q(x) to I_D (con't)

- Then ID is determined as

$$I_D = WC_{ox}[V_{GS} - V(x) - V_{TH}]\mu_n \frac{dV(x)}{dx}$$

- Because I_D is constant along to channel (that is, invariant to x),

$$[V_{GS} - V(x) - V_{TH}]\mu_n \frac{dV(x)}{dx}$$: independent to x

- Using this feature,

$$\int_{x=0}^{x=L} I_D\, dx = L \times I_D = \int_{V(x)=0}^{V(x)=V_{DS}} \mu_n C_{ox} W[V_{GS} - V(x) - V_{TH}]\, dV$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}\left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right]$$

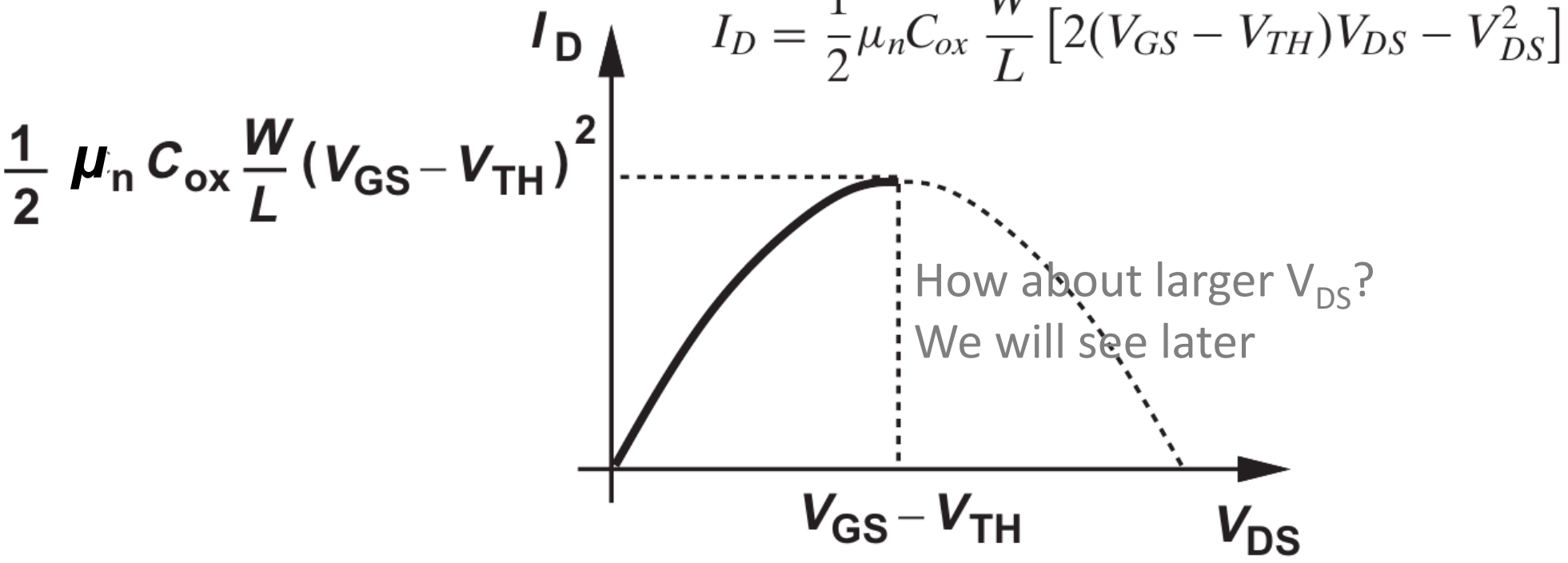
- where the maximum is

$$I_{D,max} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$

I_D vs. V_{DS} for given V_{GS}

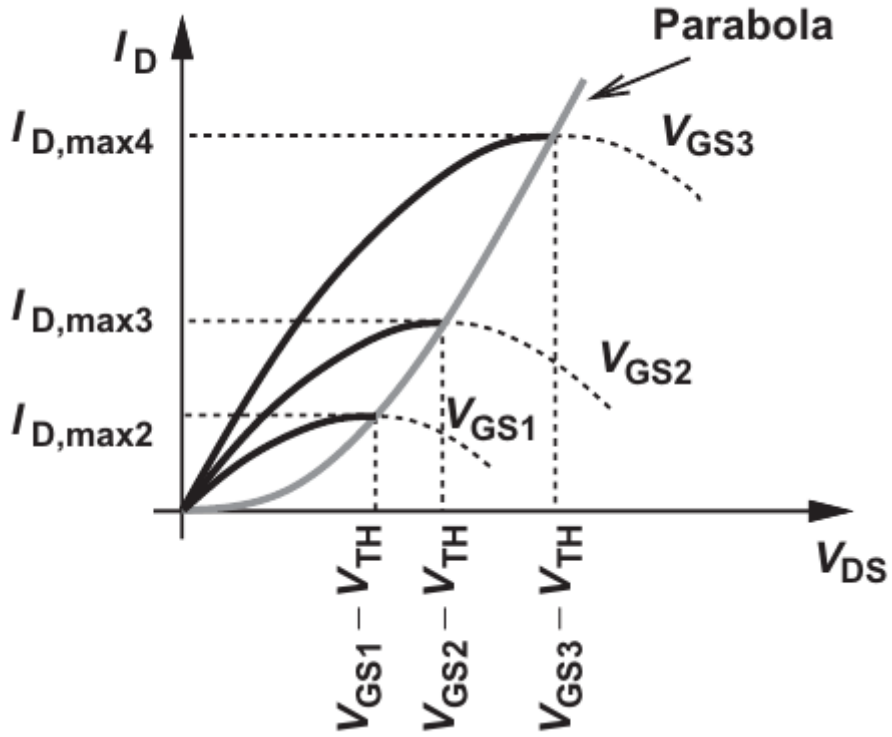
- Note that ID can be fully characterized with two node voltages (V_{GS}, V_{DS}), not only one of them.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS}-V_{DS}^2\right]$$



Example 6.3

Plot the I_D-V_{DS} characteristics for different values of V_{GS}.



Approximation for Linearization

- If $V_{DS} << 2(V_{GS}-V_{TH})$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$$



$$I_D \approx \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})V_{DS}$$

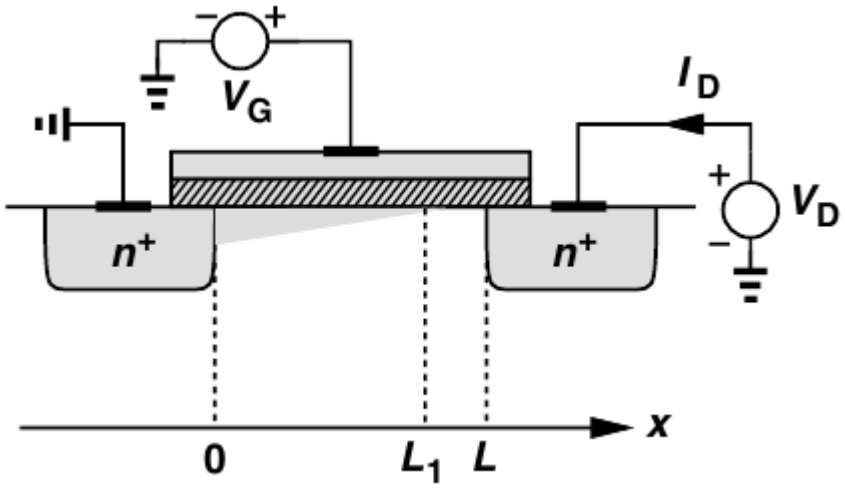
- Then, it can be considered as a resistor for V_{DS} variation. That is, $R_{on} = V_{DS}/I_D$ of

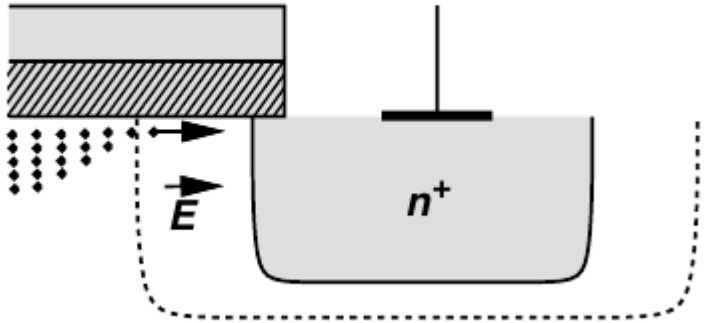
$$R_{on} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})}$$

You remember MOSFET as variable resistor?

Recall that

- Channel pinches off when $V_{DS} > V_{GS} - V_{TH}$





MOSFET at Higher V_{DS}

- Channel Pinches off

$$\int_{x=0}^{x=L_1} I_D\, dx = \int_{V(x)=0}^{V(x)=V_{GS}-V_{TH}} \mu_n C_{ox} W[V_{GS} - V(x) - V_{TH}]\, dV$$

- Two region**:**
  Triode region (=Linear region) vs. **Saturation region**

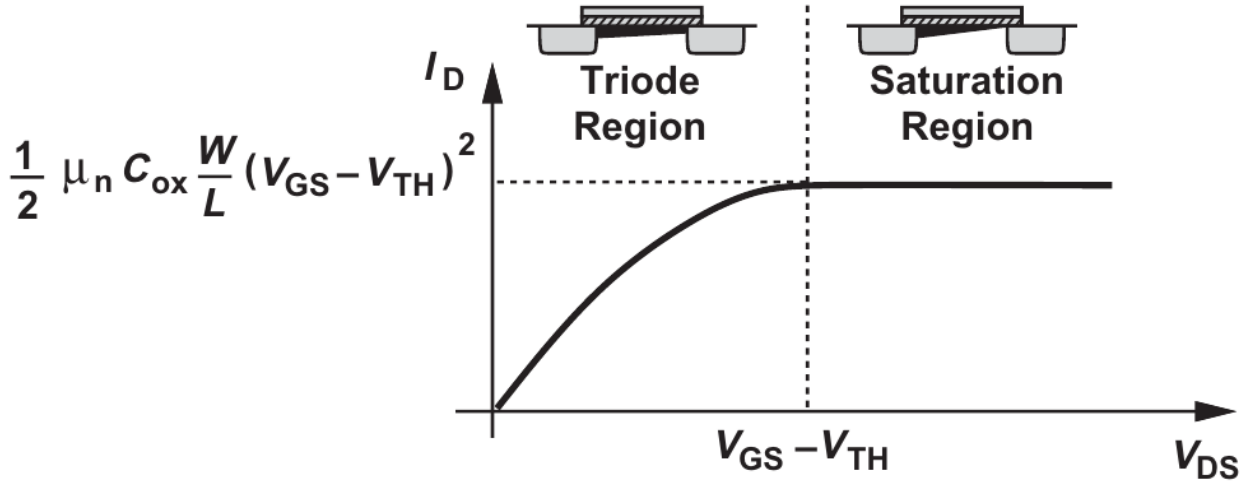
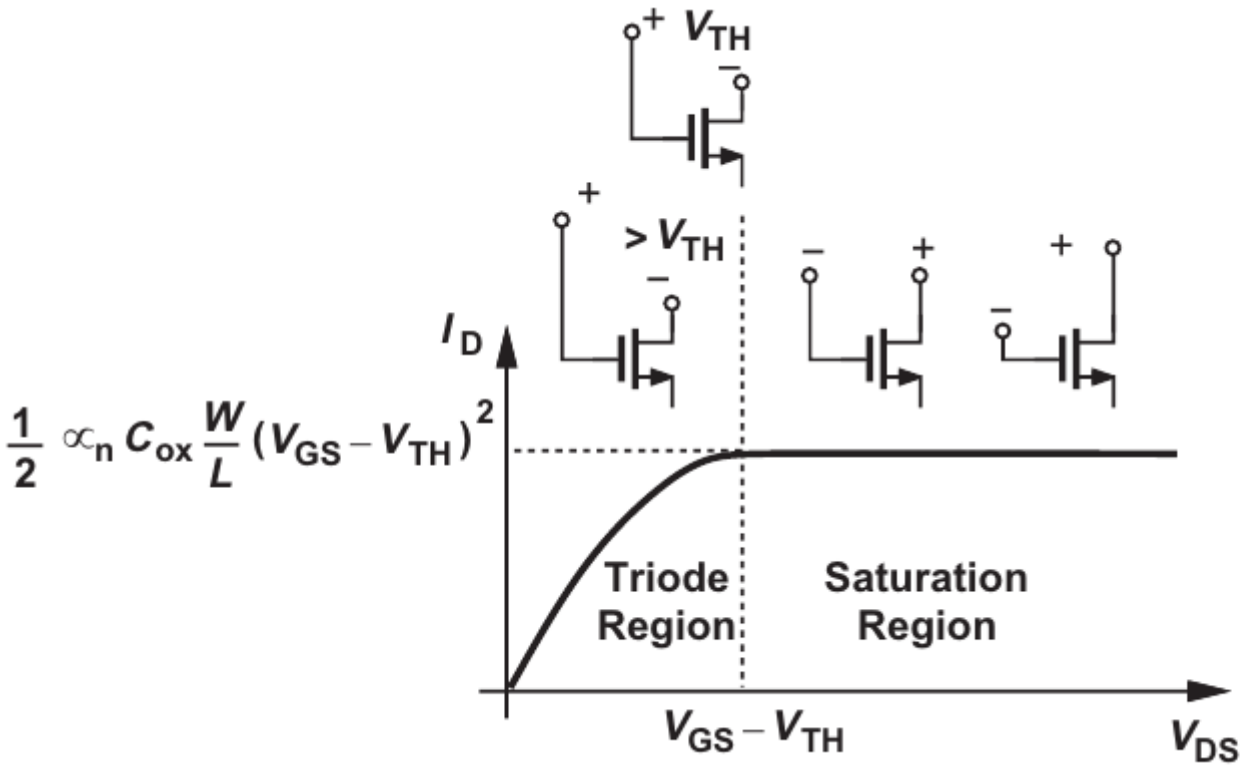
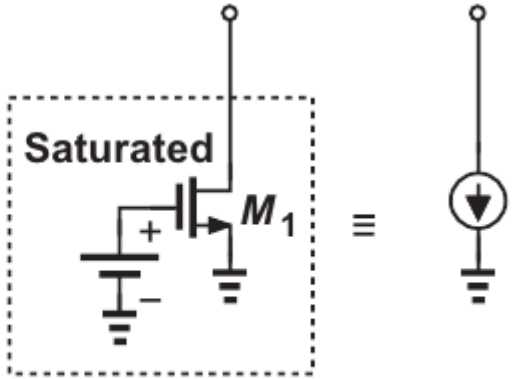


Illustration of Triode and Saturation Regions





Example 6.6

Calculate the bias current of M_1 in Fig. 6.23. Assume $\mu_n C_{ox} = 100\,\mu\text{A/V}^2$ and $V_{TH} = 0.4\,\text{V}$. If the gate voltage increases by 10 mV, what is the change in the drain voltage?

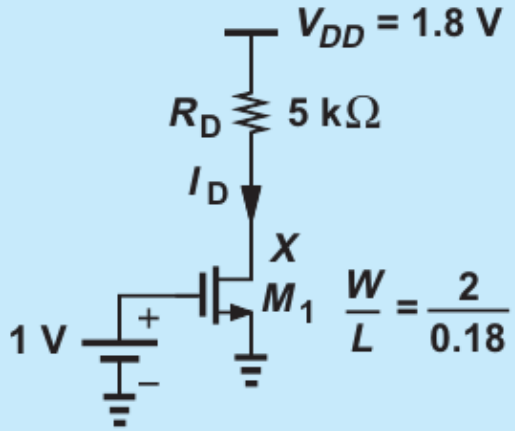
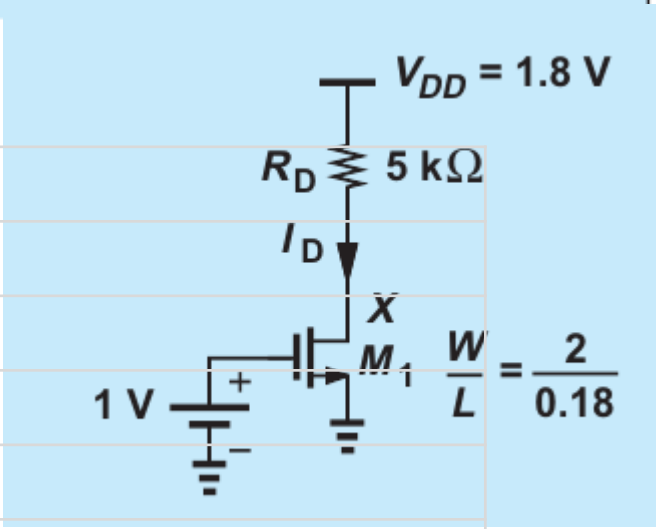


Figure 6.23 Simple MOS circuit.

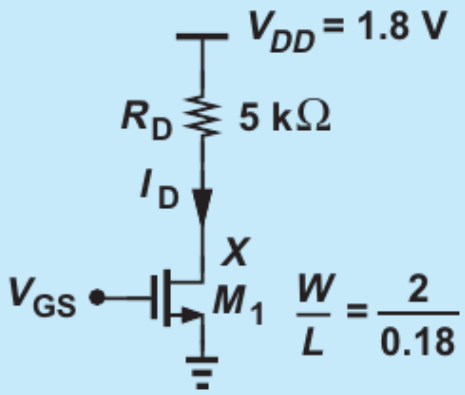
Example 6.7

Determine the value of W/L in Fig. 6.23 that places M_1 at the edge of saturation and calculate the drain voltage change for a 1-mV change at the gate. Assume $V_{TH} = 0.4\,\text{V}$.



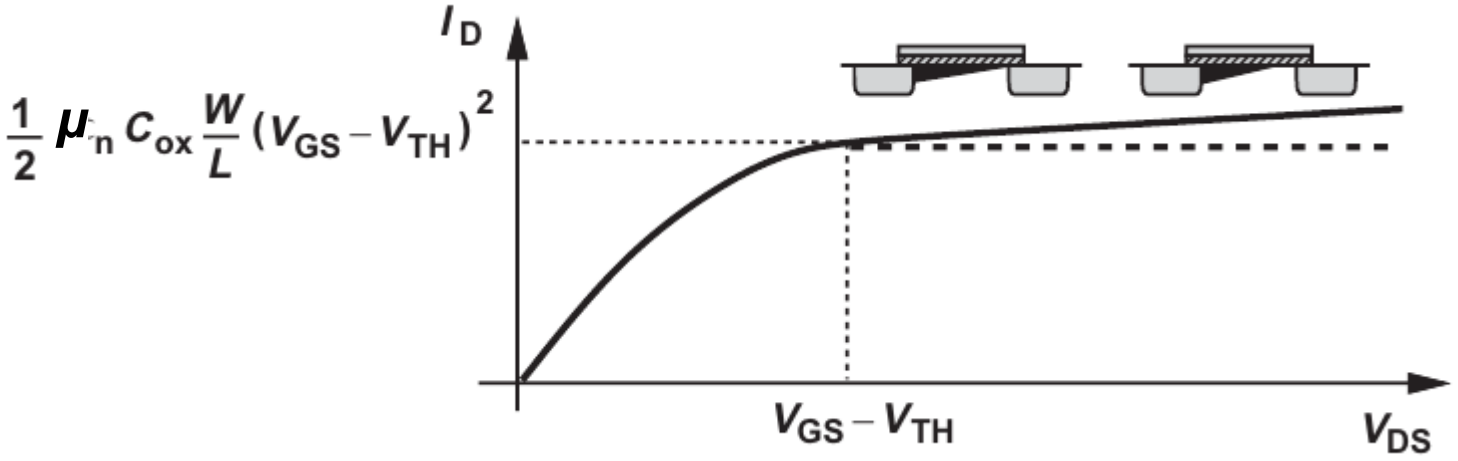
Example 6.8

Calculate the maximum allowable gate voltage in Fig. 6.24 if M_1 must remain saturated.



Channel Length Modulation

- It is observed that the point at which the channel vanishes in fact moves toward the source as the drain voltage increases.
- The effective channel length, L_1, is decreased to increase current.
- That is, in fact, I_D increases with V_{DS} even in the saturation region. ➔ Similar to Early effect in BJT

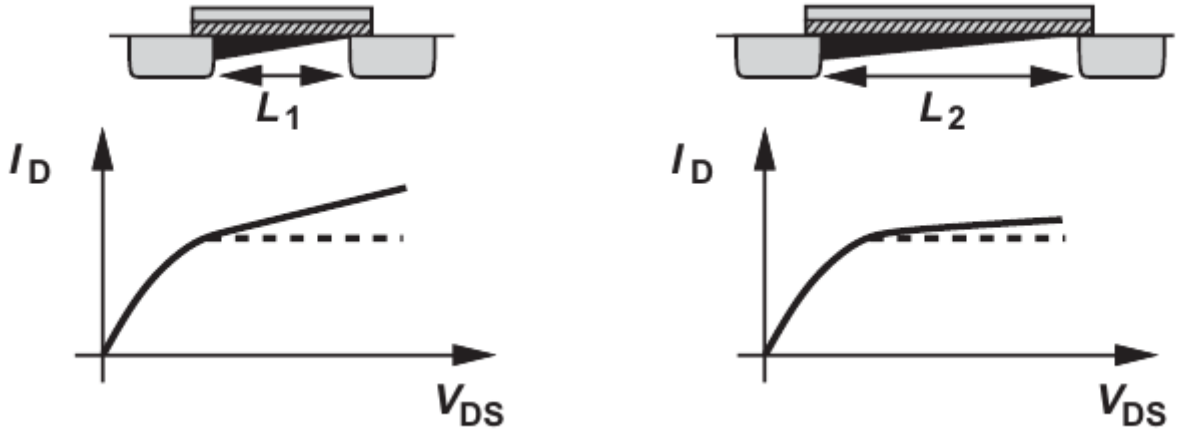


Accounting Channel Length Modulation

- λ is channel-length modulation coefficient

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

- Channel length modulation vs. L



Example 6.9

A MOSFET carries a drain current of 1 mA with $V_{DS} = 0.5$ V in saturation. Determine the change in I_D if V_{DS} rises to 1 V and $\lambda = 0.1\ \text{V}^{-1}$.

Summary

- $V_{GS} < V_{TH}$: cut off or turn off ➔ $I_D = 0$
- $V_{GS} > V_{TH}$
  - $V_{DS} < V_{GS} - V_{TH}$: Linear or triode mode ➔ $I_D =$
  - $V_{DS} > V_{GS} - V_{TH}$: Saturation mode ➔ $I_D =$

